Conformal invariance and anisotropic propagation of light in special relativity

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When conformal invariance was first introduced into physics by Cunningham and Bateman [1], it became clear that there could be a new Special Relativity, with a space-time such that its metric is invariant under the conformal group. The interest in conformal symmetry reappeared several times since then and the extended relativity theories which allow the invariance with respect to conformal transformations of the metric have been introduced in different physical contexts.

Usually, when the conformal symmetry of Minkowskis space is used instead of Poincare's symmetry, the assumption that the form of the metric changes by a conformal factor is *imposed* like as it is assumed in ordinary Special Relativity that the metric does not change. In the present paper, we show that the conformal invariance of the metric *arises* naturally in special relativity with anisotropic propagation of light. The assumption of the light speed anisotropy together with the group property for the transformations between inertial frames and the correspondence principle (correspondence of the space coordinate transformations to the Galilean transformations in the limit of small velocities is meant) inevitably leads to the transformations which do not leave the interval between two events invariant but change it by a conformal (scale) factor. It should be also noted that the coordinates normal to the direction of relative motion are also subject to the scale transformations so that the assumption commonly used in similar derivations that those coordinates do not transform may be not valid here. To derive the transformations between different inertial frames the Lie group theory apparatus is used and the two variants of the theory are developed. In one variant, the light anisotropy is treated as the basic nature law so that the anisotropy parameter is assumed to be the same in all inertial frames. In another variant, the anisotropy is considered to be a result of motion with respect to a preferred frame, in which the speed of light is isotropic, and relation of the anisotropy parameter to the velocity with respect to a preferred frame is obtained.

The transformations derived within this framework differ from the "generalized" Lorentz transformations which have been repeatedly derived and discussed in the literature in the context of the light speed anisotropy (see, e.g., [2]). Those derivations may differ in the first principles used (although the round-trip light principle and the linearity assumption are commonly present) but the resulting transformations are, in fact, those obtained from the Lorentz transformations by a change of space-time coordinates from "standard" to "non-standard" synchronization (see, e.g., [3]). However, such generalized Lorentz transformations are inconsistent in that they do not turn into Galilean transformations in the limit of small velocities but contain additional terms including the synchronization parameter and light speed – it is evident that there is no place for the issues of synchronization and light speed in the framework of the Galilean kinematics.

References

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