What is the phenomenon that keeps an infinite memory for the fluctuactions in the conduction current

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If the electron acceleration $a_{\rm ZPF}$ due to the nonrenormalized zero-point field (ZPF) of stochastic electrodynamics (SED) is introduced in the Fokker-Planck equation accounting for electron-electron acceleration ($e-e$ FP), there is always a small interval δv of speed v starting from v_1 where the two collision frequencies $\nu_1(v)$ and $\nu_2(v)$ appearing in the $e-e$ FP are both proportional to $1/v$, corresponding to the threshold of runaways. Both diffusion and drift in the v space almost vanish in the small δv where $\nu_2(v) = B\nu_1(v) = BK/v$. The Green's solution $p_0(v,\tau|v_1)$ [or a pimple on $p_0(v,\tau\to\infty)$] is almost crystallized, being $\propto \tau^{-\varepsilon}$ with $0.003 \le \varepsilon \le 0.007$. There is therefore a process of reconstruction of a fluctuaction occurring in δv, and that fluctuaction decays with a power law with such a small exponent that its memory is practically infinite.

I. INTRODUCTION

The decay time of the direction $\hat{\mathbf{v}} = \mathbf{v}/v$ of the electron velocity $\mathbf{v}$ in the conduction current is of the order of the mean free flight, i.e., $t_d \simeq 10^{-13}$ s. The average decay of $v = |\mathbf{v}|$, is given by the preceding result times $2M/m_*$, where M and m_* are the atomic mass and the electron effective mass, respectively. For instance, in Si such decay time is $t_d(v) \simeq 10^{-8}$ s. More precisely, let us consider free electrons in a uniform medium and denote $p = p(\mathbf{v}, t+\tau|t)$ the transition (or conditional) probability density in time for an electron to have the velocity $\mathbf{v}$ at time $t+\tau$ starting from the initial condition $p = p(\mathbf{v}, t)$ at time t. In a steady-state, ergodic stochastic process, the ensemble (or time) average $\langle p(\mathbf{v}, t+\tau|t)\rangle = p(\mathbf{v}, \tau)$ no longer depends on t. In the presence of an acceleration $\mathbf{a} = e\mathbf{E}/m_*$ (where $\mathbf{E}$ is an external electric field, e and m_* the charge and the effective mass, respectively, of an electron), p is usually expanded in Legendre polinomials P_l truncated after two terms $p(\mathbf{v},\tau) = p_0(v,\tau) + \hat{\mathbf{v}}\cdot\hat{\mathbf{a}}p_1(v,\tau)$. Substituting this p_1 approximation into the Boltzmann equation where electron-electron ($e-e$) interactions are neglected one obtains [1] the standard Fokker-Planck equation (FP)

$$\frac{\partial p_0}{\partial \tau} = \frac{m_*}{Mv^2}\frac{\partial}{\partial v}\left[v^3\overline{p}_0\nu(v)\right] + \frac{a^2}{3\,v^2}\frac{\partial}{\partial v}\left[\frac{v^2}{\nu(v)}\frac{\partial p_0}{\partial v}\right], \quad (1)$$

where $\nu(v)$ the electron collision frequency with the lattice, and

$$\overline{p}_0 = p_0 + \frac{kT}{m_* v}\frac{\partial p_0}{\partial v}\ , \quad (2)$$

the so called "Davidov approximation" (k denotes the Boltzmann constant, T the absolute temperature). Once found the isotropic component p_0, the anisotropic component p_1 is given by $p_1 = -[a/\nu(v)]\partial p_0/\partial v$, and all the transport quantities (like drift velocity, diffusion, relaxation, etc.) can be calculated. Some authors, and more extensively Stenflo [1], solved Eq. (1) when $T = 0$ and

$$\nu(v) = Kv^n\ , \quad (3)$$

so that Eq. (1) becomes

$$\frac{\partial p_0}{\partial \tau} = \frac{m_*}{Mv^2}\frac{\partial}{\partial v}\left(Kv^{3+n}\,p_0\right) + \frac{a^2}{3v^2}\frac{\partial}{\partial v}\left(\frac{v^{2-n}}{K}\frac{\partial p_0}{\partial v}\right)\ . \quad (4)$$

Transition probability density solutions for Eq. (4) can then be derived from Stenflo's results [1]. Such time-dependent Green's solution $p(v,\tau|v_0)$ means that if, at the initial time, all the electron speeds have the value v_0, the delta function drifts from v_0 towards the equilibrium most probable value, at the same time diffusing in the speed space, until it becomes the steady-state distribution $p_0(v) = p_0(v,\tau\to\infty)$, which is independent of v_0. The same behavior is common to a "pimple", due to a noise fluctuaction on $p_0(v)$. For $n > -1$, the difference $F(v,\tau) = p_0(v,\tau|v_0) - p_0(v)$ turns out to be expressed by a series $\sum_i f_i \exp(-\tau/\tau_i)$. If we retain only the first term, which is the most important because it has the longest decay, we may write $F(\tau) \propto \exp[-\tau/\tau_0(n)]$, meaning that

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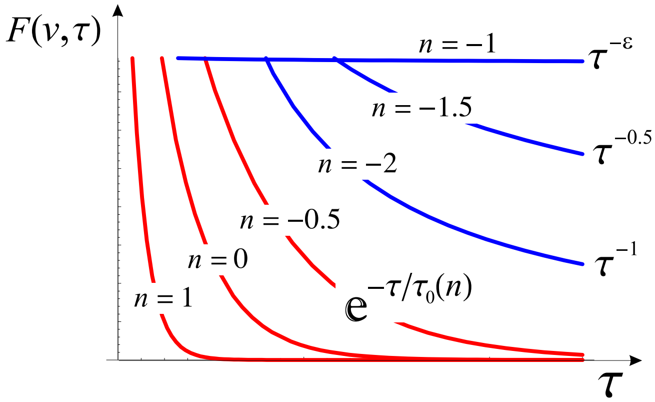

FIG. 1: $F(\tau) = p_0(v,\tau|v_0) - p_0(v)$ vs τ when the collision frequency is of the kind $\nu_2 = Kv^n$ and for different n values. $p_0(v,\tau|v_0)$ denotes the transition (or conditional) probability density to have a normalized distribution function $p_0(v,\tau)$ in v at time τ for electrons started at $\tau = 0$ with a delta distribution centred at v_0. Moreover, $p_0(v) = p_0(v,\infty|v_0)$.

the relaxation due to both drift and diffusion is characterized by a single time constant $\tau_0(n)$, whose value increases with the decrease of n, as shown in Fig. 1. For $n \leq -1$ there is a drastic change in the relaxation that becomes characterized by a power law. The diffusion in the speed space vanishes, and only a drift towards higher speeds remains, so that $p_0(v,\tau \to \infty|v_0) = p_0(v) \to 0$, and $F(v,\tau) = p_0(v,\tau|v_0)$. The free electrons tend to become collisionless and to acquire increasingly higher speeds: they are in runaway conditions. For $n \leq -2$, for long times, it is $F(\tau) \propto \tau^{n+1}$. For $n = -1$, condition at the threshold of runaways, it is [1]

$$p_0(\tau) \propto \tau^{-\varepsilon} \quad \text{with} \quad \varepsilon = 3\left(1 - \frac{m_*}{M}\frac{K^2}{a^2}\right) , \qquad (5)$$

where K is defined by Eq. (3). The ε expression is paradoxical because it starts from 3 for $a^2 \to +\infty$, vanishes for $a^2 = K^2 m_*/M$, then becomes negative for smaller a^2 values, tending to $-\infty$ for $a^2 \to 0$.

The paradox is solved in Sec. II by the introduction of the zero-point field (ZPF) of stochastic electrodynamics (SED). Such ZPF is equal to the one of quantum electrodynamics (QED), although it is nonrenormalized, i.e., considered as real, ubiquitous, and solving the stability of the atoms. Here we show that even the Fermi energy can be derived in a classical way from the ZPF, i.e., in an alternative way with respect to quantum mechanics (QM).

In Sec. III we exploit the reduction of the nonlinear Boltzmann equation with electron-electron interactions to a Fokker-Planck equation ($e-e$ FP), found in two previous works [2, 3]. Introducing the ZPF in the $e-e$ FP, the two collision frequencies appearing in it turn out to be both proportional to $1/v$, i.e.

$$\nu_2(v) = B\nu_1(v) = BK/v \qquad (6)$$

(where B and K are constants) in a small v interval δv starting from a value v_1 dependent on the electron number density N. Equation (6) corresponds to the threshold of runaways, at which the diffusion in the v space almost vanishes and becomes ballistic in the configuration space. The drift in the v space does not vanish generally, but with a^2_{ZPF} in the $e-e$ FP, also the drift in the v space almost vanishes (only in the δv starting from v_1). Then the Green's solution, or a pimple in $p_0(v,\tau \to \infty)$, becomes almost crystallized so that $p_0(v_0,\tau) \propto \tau^{-\varepsilon}$ with a very small ε value ($0.003 \leq \varepsilon \leq 0.007$).

We conclude in Sec. IV, showing an application of the above mechanism of reconstruction of a fluctuaction on $p_0(v,\tau \to \infty)$.

II. SOLUTION OF THE PARADOX AND ALTERNATIVE DERIVATION OF THE FERMI ENERGY BY THE ZPF OF SED

The paradoxical expression of Eq. (5), where ε can become negative and diverge for $a^2 \to 0$, is in a certain sense similar to the problem relevant to the stability of the atoms. Being confined, the motion of an electron around a nucleus is accelerated, and it has therefore to radiate e.m. power. The problem has not qualitatively been solved by the (postulated) Schroedinger equation, but by the introduction of the spin motion according to the solution of the Dirac equation, i.e., as a motion with the speed of light of an almost point-like particle along a circular orbit having the Compton radius [4]. This motion can realistically be justified as due to self-reaction, and eliminating special relativity (SR) at the subparticle level in order to have finite values for both the mass and the radiation [5]. Then SR can be derived because one usually considers, as the particle velocity, the one of the ideal centre around which the electron revolves, and not the real one at the speed of light [6]. Furthermore, the radiation due to the spin "gyration" (or revolution) of all the particles of the universe, progressively red-shifted because of the universe expansion, turn out to have a power-spectral density proportional to the cube of $\omega = 2\pi f$, where f is the frequency [6, 7]

$$S_{\mathrm{ZPF}}(\omega) = \sum_i \frac{N_i}{H} P_{\mathrm{rad}\,i} \frac{\omega^3}{\omega_{si}^4} = \frac{\hbar\omega^3}{2\pi^2 c^3} = \frac{2hf^3}{c^3} , \qquad (7)$$

the second side being written in terms of the Hubble constant H, the average number density N_i in the universe of the i-th spinning particle having spin pulsation ω_{si} and radiated power $P_{\mathrm{rad}\,i}$, while the third and fourth sides are written in the usual terms of the Planck constant h and the speed of light c. The spectrum (7) is equal to the ZPF of QED, although it is renormalized in QED, mainly because it is divergent for $\omega \to \infty$. However, the ZPF cannot be renormalized in presence of gravity (i.e., in a Riemannian space), and this is a big trouble for QED. On the contrary, it has always taken in a real sense, hence nonrenormalized, in SED where some upper cut-off has been artificially introduced. To a better reason, in the present approach, we can denote as SED with spin, there is the natural cut-off expressed by the maximum spin frequency ω_{siM} (i.e., the one radiated by the particle having the smallest Compton radius R_{sim} because $\omega_{siM} = c/R_{sim}$).

Having a real, ubiquitous ZPF, the stability of the atom is immediately explained because any charged oscillator, as an electron revolving around a nucleus, absorbs power from a stochastic e.m. field so that an electron radiates what it absorbs on an average. Assuming for simplicity a circular orbit with radius r and denoting e and v the electron charge and speed, respectively, it is, with the use of Eq. (7)

$$P_{\rm abs} = n\frac{2}{3}\frac{e^2}{m}\pi^2 S_{\rm ZPF}(f) = \frac{8\pi^2}{3}\frac{e^2}{m}\frac{h}{c^3}\left(\frac{v}{2\pi r}\right)^3 , \quad (8)$$

where m denotes the electron mass, and $n = 2$ is the number of harmonic oscillators necessary to reproduce a circular motion. The radiated power, in absence of SR, is given by the Larmor expression

$$P_{\rm rad} = \frac{2}{3}\frac{e^2}{c^3}\left(\frac{v^2}{r}\right)^2 . \quad (9)$$

Equating Eq. (8) to Eq. (9) we obtain $mvr = h/(2\pi)$, which is the Bohr condition for hydrogen's fundamental state.

Now the sizes of the atoms and the periods of revolution around the nuclei can be derived from the Bohr condition that, in turn, depends on $S_{\rm ZPF}(f)$. If an atom is at rest in a frame F, and another atom in F', each observer measures the same size and the same period of revolution for its atom at rest if $S_{\rm ZPF} = S'_{\rm ZPF}$. Now, Boyer [8] has shown that $S_{\rm ZPF} = S'_{\rm ZPF}$ if the coordinates in frame F are related to those in frame F' via Lorentz transformations. In other words, the $S_{\rm ZPF}(f)$ expressed by Eq. (7) is Lorentz invariant (and it is the only one spectrum having that property). Consequently, also atomic lengths and frequencies (that depend on $S_{\rm ZPF}$) in frames F and F' are related by Lorentz transformations.

We have therefore found SR that could also be inferred by the gyrating electron (producing the improperly called spin). Indeed, let us consider an electron whose centre of revolution O is at rest in frame F' having therefore a gyration period P' (for F') given by $P' = 2\pi R_s/c$. For the frame F, the centre O moves with $\mathbf{v}$ perpendicular to the plane α of gyration. Since for F the electron moves with c along a helix, the period P to traverse a pitch is longer. Precisely, the component $c_\perp$ on α of its velocity is $c_\perp = \sqrt{c^2 - v^2}$, so that the period for F is

$$P = \frac{2\pi R_s}{c_\perp} = \frac{2\pi R_s}{c(1 - v^2/c^2)^{1/2}} = \gamma P' , \quad (10)$$

just as given by SR, here derived from Galilean kinematics by the Pythagorean theorem. We clearly see that SR has to be applied to the centre O of the electron gyration (and not to the real motion at the speed c).

The spin (or better "gyration") motion of the electron at the speed of light allows not only the derivation of SR, of the ZPF spectrum (7), and the Bohr condition, but also the derivation of the Schroedinger equation for a single particle [9], and for many indistinguishable particles [10], as acknowledged by the director of Nature in "News and view" [11]. With the same method, even a generalization of the Schroedinger equation has been obtained [12], whose corrective terms turn out to be $\simeq 1\%$ of the Lamb shift [13]. Many other results of QM has been obtained by pure SED (i.e., without spin) by Boyer [14] (and references therein). Two novelties beyond QM are the origin of the high-energy tail of the cosmic rays spectrum [15, 16], and the explanation of some anomalies related to the neutrino mass [7].

We show here that even the Fermi energy U_F can be obtained without using the Fermi-Dirac statistics. Using the above derived Bohr condition put in the better form $m\langle v^2\rangle^{1/2}\langle r^2\rangle^{1/2} = \hbar$ we obtain

$$U_F = \frac{m}{2}\langle v_0^2\rangle = \frac{\hbar^2}{2m\langle r^2\rangle} , \quad (11)$$

where $\langle r^2\rangle^{1/2}$ is the equivalent amplitude of an oscillation corresponding to a single collision (or scattering). In fact, the ZPF at high frequencies is very intense, but without collisions the speed increase acquired in a half oscillation is lost in the subsequent half collision. On the contrary, let us consider a ZPF wave train, with frequency f, which impinges on an electron having speed $\langle v_0^2\rangle^{1/2}$ at a distance $\langle r_0^2\rangle^{1/2}$ from the collision point. If $(\langle v_0^2\rangle/\langle r_0^2\rangle)^{1/2} \simeq f$ and the scattering angle θ is $5\pi/6 < \theta < \pi$, the electron receives a strong impulse that keeps it close to the Fermi energy. For two free electrons with mutually opposite spins, one close to an atom, and the other close to another, adjacent atom, it is roughly

$$\langle r^2\rangle^{1/2} \simeq \frac{1}{4N^{1/3}} , \quad (12)$$

where N denotes the atom number density.

Substituting Eq. (12) into Eq. (11), we obtain

$$U_F = \frac{\hbar^2}{2m}(64N)^{2/3} , \quad (13)$$

which has the same dependence on $\hbar$, m, N as the exact $U_F^{\rm ex}$, whose coefficient inside the round bracket is $6\pi^2$, close to our 64. The corresponding acceleration is

$$\langle a^2_{\rm ZPF}\rangle_t = \frac{\langle v_0^2\rangle_t^2}{\langle r^2\rangle} = \left(\frac{64\hbar N}{m_*^2}\right)^2 \quad (14)$$

which, once substituted for a^2 into Eq. (5), yields $Ma^2_{\rm ZPF} \gg m_* K^2$, so that $\varepsilon \simeq 3$. Not only the paradox is eliminated, but also the use of Eq. (14) in the FP accounting for $e-e$ interactions leads to a new $\varepsilon \simeq 0.005$, thus implying an extremely slow decay of $p_0(v,\tau)$.

III. FOKKER-PLANCK EQUATION WITH ELECTRON-ELECTRON INTERACTIONS ($e-e$ FP) AND ITS SOLUTIONS WITH $a^2 \simeq a^2_{\rm ZPF}$

In a previous paper [2], the nonlinear Boltzmann equation with electron-electron interactions has been reduced to a Fokker-Planck equation ($e-e$ FP). The used method was partially analytical and partially numerical and the necessary use of modern computers explains why hundred years have been required for such an achievement. The important fact is that the result (the Fokker-Planck equation) is expressed in a compact analytical form.

In a subsequent paper [3], the method has been applied to doped silicon and somewhat improved by exploiting axial symmetry and using quantum physics for the calculations of cross-sections (hence collision frequencies).

The resulting $e-e$ FP is given by Eq. (54) of Ref. 3 that we report here in a convenient version,

$$\frac{\partial p_0}{\partial t} = \frac{1}{v^2}\frac{\partial}{\partial v}\left\{ v^3 \left[p_0(v,t) + \frac{kT}{m_* v}\frac{\partial p_0}{\partial v}\right] \nu_1(v) \right\} + \frac{a^2}{3\,v^2}\frac{\partial}{\partial v}\left[\frac{v^2}{\nu_2(v)}\frac{\partial p_0}{\partial v}\right], \quad (15)$$

where $\nu_1(v)$ is an equivalent collision frequency derivable from Eq. (54) of Ref. 3 and given by

$$\nu_1(v) = \frac{1}{3}A_0(v) + \frac{m_*}{M}\,\nu_m(v), \quad (16)$$

ν_m denoting the electron collision frequency for momentum transfer with ions and semiconductor lattice via acoustic and optical phonons (see Appendix A of Ref. 3), and $A_0(v)$ is expressed by Eq. (37) of Ref. 3.

Similarly, we derive from Eq. (54) of Ref. 3

$$\nu_2(v) = \langle \nu_{me}\rangle(v) + \nu_m(v), \quad (17)$$

where $\langle \nu_{me}\rangle(v)$ is the average value of the electron-electron collision frequency for momentum transfer, given by Eq. (39) of Ref. 3.

In steady-state conditions, i.e., for $\partial p_0(v,t)/\partial t = 0$, the solution of Eq. (15) is given by Eq. (58) of Ref. 3, which is a kind of Chapman-Cowling-Davydov expression

$$p_0(v) = \exp \int_0^v - \frac{m_* v\, dv}{kT + m_* a^2 (3\nu_1\nu_2)^{-1}} \;. \quad (18)$$

The effect of the acceleration $\mathbf{a}$ is to produce an equivalent temperature T_{eq}, or $\langle m_* v^2/2\rangle = 3kT_{eq}/2$. Since the square a^2 appears in both Eqs. (15) and (18), the effect of a high frequency oscillating field $\mathbf{E}$ is equivalent to a D.C. field provided we substitute $\langle a^2\rangle$, averaged over a period, for a^2 in Eqs. (15) and (18).

According to Sec. II we now write $\mathbf{a} = \mathbf{a}_{\rm D.C.} + \mathbf{a}_{\rm ZPF}$ with $\langle \mathbf{a}_{\rm ZPF}\rangle_t = 0$, so that $\langle a^2\rangle_t = \mathbf{a}^2_{\rm D.C.} + \langle \mathbf{a}^2_{\rm ZPF}\rangle_t \simeq \langle a^2_{\rm ZPF}\rangle_t$ because the second term is much larger than the first (due to a D.C. field). With the value given by Eq. (14), the steady-state probability density (6) becomes similar to the Fermi-Dirac's, and there is always

N $\rm m^{-3}$	a $\rm ms^{-2}$	$10^{-5}v_1$ $\rm ms^{-1}$	$10^{-3}\delta v$ $\rm ms^{-1}$	$p_0(v_1)$	$10^3\varepsilon$
10^{20}	6.3×10^{18}	4.23	1.01	0.55	7
10^{21}	1.2×10^{20}	4.25	1.02	0.54	6
10^{22}	2.9×10^{21}	4.32	1.03	0.53	6
10^{23}	8.13×10^{22}	4.38	1.04	0.52	6
10^{24}	1.8×10^{24}	4.39	1.05	0.49	5
10^{25}	1.85×10^{25}	4.40	1.04	0.47	4
10^{26}	1.2×10^{26}	4.40	1.03	0.45	3

TABLE I: Values of the fundamental parameters vs the concentration N of free electrons. The electron acceleration a is mainly due to the ZPF, i.e., $a \simeq a_{\rm ZPF}$. A fundamental result is the exponent ε of the decay time in Eq. (23).

(i.e., for any N value) a small interval δv (starting from a speed v_1) such that

$$\nu_2(v) = B\nu_1(v) = BK/v \;, \quad (19)$$

where B is a constant. In the effective δv interval [where Eq. (19) holds] the $e-e$ FP for $T=0$ reduces to

$$\frac{\partial p_0}{\partial t} = \frac{1}{v^2}\frac{\partial}{\partial v}\left(K v^2\, p_0\right) + \frac{a^2}{3v^2}\frac{\partial}{\partial v}\left(\frac{v^3}{BK}\frac{\partial p_0}{\partial v}\right) \;, \quad (20)$$

which is similar to the standard FP, Eq. (4) with $n=-1$.

The solution of this partial differential equation has been given by Stenflo [1] and reads

$$p_0(v,\tau_r) = \frac{v^\alpha}{\tau_r}\exp\left(-\frac{v}{\tau_r}\right)\int_0^\infty du\, p_0(u,0)u^{-\alpha} \times \exp\left(-\frac{u}{\tau_r}\right) I_{2\alpha+4}\left(\frac{2\sqrt{vu}}{\tau_r}\right) \;, \quad (21)$$

where I_p is the modified Bessel function of the first kind, and, with our notations and constants

$$\alpha = -1 - \frac{3BK^2}{2a^2} \;; \qquad \tau_r = \tau a^2/3K \;. \quad (22)$$

For large v and τ values, Eq. (21) reduces to

$$p_0(v,\tau) = \frac{A v^{-3BK^2/a^2}}{\Gamma(\varepsilon)\tau^\varepsilon}\int_0^\infty du\, u^2\, p_0(u,0) \;, \quad (23)$$

where Γ is Euler's gamma function, A a constant, and

$$\varepsilon = 3(1 - BK^2/a^2) \;. \quad (24)$$

The ε values can in general be either positive, nil, or negative, tending to $-\infty$ for $a^2 \to 0$. However, and this is the most important consequence of not having neglected the ZPF, with $a^2 \simeq a^2_{\rm ZPF}$ it is $0.003 \le \varepsilon \le 0.007$, as shown in Table I. The time decay almost vanishes for large v and τ_r values. On the other hand, the transition from the initial time decay expressed by Eq. (21) is always very slow so that the boundary conditions in the speed space have a very little influence. This fact is very important because Stenflo solved Eq. (20) considering Eq. (9) as valid in all the interval $0 \le v \le \infty$, while in our case it is satisfied in δv only and the boundary conditions in v space are different. Nevertheless, for very small ε values $p_0(v,\tau)$ depends on τ so weakly in δv that we have a crystallization of a fluctuation and we can have any boundary conditions because there is practically no time evolution.

In order to turn Eq. (23) into a transition probability $p_0(v,\tau|v_0)$ the initial probability density $p_0(u,0)$ appearing in Eq. (23) has to be concentrated at a single v_0 value. Moreover, in order to be normalized $1 = \int_0^\infty du\, 4\pi u^2 p_0(u,0)$ it must take the expression

$$p_0(u,0) = (4\pi {v_0}^2)^{-1}\, \delta(u - v_0)\ , \qquad (25)$$

where δ denotes the Dirac's delta function. Substituting Eq. (25) into Eq. (23), and using Eq. (22) with τ for τ_r, we obtain

$$p_0(v,\tau|v_0) = \frac{A v^{\varepsilon-3}(3K/a^2)^\varepsilon}{4\pi\Gamma(\varepsilon)\tau^\varepsilon}\ , \qquad (26)$$

which is independent of v_0. As said, Eq. (23), hence Eq. (26), is valid for sufficiently large v and τ, i.e., for $v > \langle v^2\rangle^{1/2}$ and $\tau \gg t_f$, where $t_f \simeq \nu_2^{-1}(v)$ is a free flight time. Looking at Fig. 2, we see that $v_1 \simeq 2.02\langle v^2\rangle^{1/2} = 4.32 \times 10^5$ ms^{-1} is larger than twice the square root of the mean square value. Moreover, the relaxation time τ_m of the high energy tail just in correspondence of v_1 turns out to be dominated by triple collisions and is of the order[17]

$$\tau_m \simeq 8.6 \times 10^{-5} s\ , \qquad (27)$$

much greater than the average time of free flight $t_f \simeq \nu_2^{-1}(v) \simeq 8.6 \times 10^{-17}$ s. The two conditions $v > \langle v^2\rangle^{1/2}$ and $\tau \gg t_f$ are therefore well satisfied. In runaway conditions there is a process that turns the exponential decay into a power law, but τ_m remains the time of the transmission of information, and also the time necessary for the relaxation of Eq. (21) to Eq. (23).

Having found that Eq. (26) is valid for $\tau \ge \tau_m \gg t_f$, and that Eq. (21) is "crystallized" for $0 < \tau < \tau_m$, we may take τ_m as the initial, starting time for τ, and extend Eq. (26) to $\tau \to 0$, provided we use $\tau_m + \tau$ for τ, thus obtaining

$$p_0(v,\tau|v_0) = \frac{A v^{\varepsilon-3}(3K/a^2)^\varepsilon}{4\pi\Gamma(\varepsilon)(\tau_m+\tau)^\varepsilon}\ . \qquad (28)$$

We can find the constant A, appearing in Eq. (28), by equating $p_0(v,0|v_0)$ to the equilibrium value $p_0(v)$ [given by Eq. (18)] with $v = v_1$ which is the beginning of the v interval δv where Eq. (9) holds. We obtain

$$p_0(v = v_1, 0|v_0) = p_0(v_1) = \frac{A(3Kv_1/a^2)^\varepsilon}{4\pi v_1^3 \Gamma(\varepsilon)\tau_m^\varepsilon}\ . \qquad (29)$$

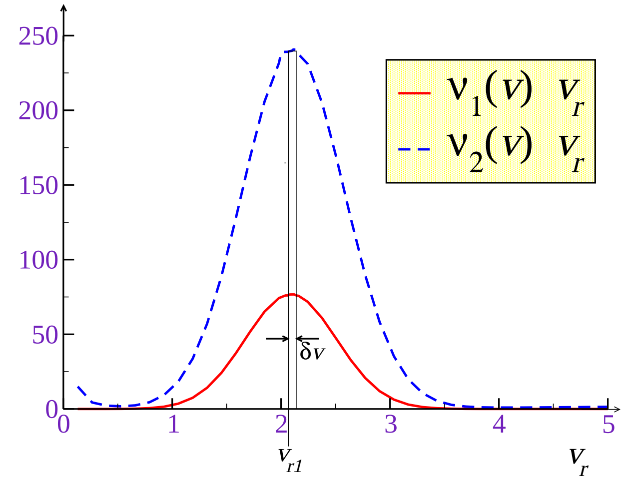

FIG. 2: Plot of the functions $\nu_1(v_r)\, v_r$ and $\nu_2(v_r)\, v_r$ (in 10^{14} s^{-1}) vs v_r, defined as $v_r = v\langle v^2\rangle^{-1/2}$. In the interval $2.0215 < v_r < 2.0263$, the two functions are constant, so that the collision frequencies $\nu_1(v)$ and $\nu_2(v)$ go as $1/v$ satisfying Eq. (19).

Finally, deriving A from Eq. (29) and substituting it into Eq. (28), we obtain

$$p_0(v,\tau|v_0) = p_0(v_1)\frac{(v_1/v)^{3-\varepsilon}\tau_m^\varepsilon}{(\tau+\tau_m)^\varepsilon}\ , \qquad (30)$$

which is the desired conditional probability density (or Green function), showing the very slow time decay because ε, as appears from Table I, is very small.

IV. CONCLUSIONS

The final result (30) shows that there is a small v interval δv where the conditional (or transition) probability $p_0(v,\tau|v_0)$ remains almost crystallized. In our calculations, performed for silicon, both δv, and its initial point v_1, are given in Table I for different values of the number density N. The result (30) is always valid because it is practically due to electron-electron $(e-e)$ interaction, so that it is independent of the lattice scattering, i.e., of the different materials. Moreover, the existence of a δv where the two collision frequencies are both proportional to v^{-1} (condition at the threshold of runaways) is due to the zero-point field (ZPF) that is universal and ubiquitous. This new phenomenon balances both the drift and the diffusion of $p_0(v,\tau|v_0)$ in the velocity space. The point is: does this new result, interesting in itself, lead to some observable consequence? The answer is positive, as shown in the companion paper, but not for drift and diffusion in the configuration space, because the fraction of electrons in δv is $\simeq 10^{-3}$. Its observation is possible only for the generalized diffusion coefficients [18] and for the noise power spectral density. In fact, in the companion paper, it is shown that the result (30) leads straightforwardly to the pure (or real, or exact) $1/f$ noise, i.e., to the one obtainable after subtracting the usually larger effect due to the material defects. In semiconductors, the residual, or pure, $1/f$ noise is equal to the one measured in the purest and quietest semiconductors. A new, never noticed N dependence is predicted, in excellent agreement with the experimental results.

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