

# The origin of the famous, pure $1/f$ noise is explained as an effect of the zero-point field acting on the free electrons of the conduction current

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The introduction of the ZPF leads to a probability density  $p_0(v)$  (where  $v$  is the electron speed) similar to the Fermi-Dirac distribution, and to a correlation function  $C_G(\tau)$  of the conductance  $G$ , which, in a small, unique  $v$  interval  $\delta v$  (where the electrons are at the threshold of runaways) decays as  $\tau^{-\varepsilon}$  with  $0.003 \leq \varepsilon \leq 0.007$ . The corresponding power spectral density turns out to be  $S_G(f) = G^2 \alpha_\varepsilon \mathcal{N}^{-1} (2\pi\tau_m)^\varepsilon f^{\varepsilon-1}$ , where  $f$  is the frequency,  $\mathcal{N}$  the total number of electrons in the considered sample,  $\tau_m$  the information transmission time, and  $\alpha_\varepsilon$  a dimensionless quantity depending on electron number density  $N$ . For the purest semiconductors,  $\alpha_\varepsilon$  that turns out to be in excellent agreement with the experimental data vs  $N$ . The above result also holds for a finite sample because the electron diffusion in the small  $\delta v$  is much more rapid than the drift velocity.

## I. INTRODUCTION

The noise whose power spectral density  $S(f)$  is roughly inversely to the frequency  $f$ , usually in a limited  $f$  range, is denoted as  $1/f$  noise. Starting from Weissman review [1], the  $1/f$  in semiconductors was considered as due to trapping-detrapping of electrons. Actually, the curves reported by Weissman [1], regarding the spectral slope of Fig. 8, the  $fS(f)$  of Fig. 9, and the  $\log S(f)$ , all vs  $f$  (which are the only plots vs  $f$ ), are very far from being a real  $1/f$  noise. This behaviour has also been predicted in a recent theory [2] strictly dedicated to semiconductors. A flattening of  $S(f)$  for  $f < f_0$  is found, in some cases with  $f_0 \simeq 10^{-3} \div 10^{-2}$  Hz much larger than the observed  $f_0 \simeq 2 \times 10^{-7}$  Hz.

Since in many cases the traps, and their  $\tau_{0s}$ , can be detected, it was the merit of Hooze's team of research [3, 4] to have subtracted their, usually main, contribution, and also the thermal noise, from the total  $1/f$  similar noise, and having shown that the remaining noise is an exact (or real, or pure)  $1/f$  noise. The residual, pure  $1/f$  noise is equal to the one for the quietest semiconductors, as found in the samples prepared by the Hooze group [3-6].

A pure  $1/f$  noise requires a fundamental theory. Such theory seemed to be the one developed by Handel [7, 8] who claimed that the  $1/f$  noise was due to low frequency photon emission by part of electrons. The current mod-

ulation seemed to be a "beating" term. But Kiss and Heszler [9] proved by rigorous QM that the beating term is zero. Moreover, the screening (cage effect) due to the considered sample, and the set-ups surrounding it, eliminate the soft photons at extremely small frequencies (up to 1/month) necessary to produce the scattering with the conduction current. Finally, the scattering with the lattice prevents the long coherence time between electrons and soft photons [1, 3, 10]. The two last criticisms apply to a recent variant of Handel's theory [11]. An interesting recent theory [12] of coupled harmonic oscillators implies, to within an approximation, an  $S(f) \propto 1/f$  for low  $f$  values. However, free electrons are evidently not harmonic oscillators. Even the electrons in atoms have a distribution function in  $r$ , for instance exponential in 1S state. Consequently, their periods of revolutions around their nuclei have wide spreadings.

The hope to have a fundamental theory for the pure  $1/f$  noise also decreased with the cumulation of experimental results. Actually, the data turned out to be widely scattered, up to four orders of magnitude, even if the results are limited to the purest semiconductors at the same absolute temperature  $T$ , as appears from Fig. 1, where for  $T \simeq 300$  K there is the majority of the data. It is clear that it is impossible to summarize the results of Fig. 1 by the Hooze's formula [3]

$$S(f)/G^2 = \alpha(T)/(\mathcal{N}f), \quad (1)$$

(where  $S(f)$  is the power spectral density of the conductance  $G$ , and  $\mathcal{N}$  the total number of charge carriers), if  $\alpha(T)$  is taken as a function of the only absolute temperature  $T$ . Nevertheless, as shown in the companion paper [13], in any conductor, and in any semiconductor, the zero-point field (ZPF) of QED but not renormalized

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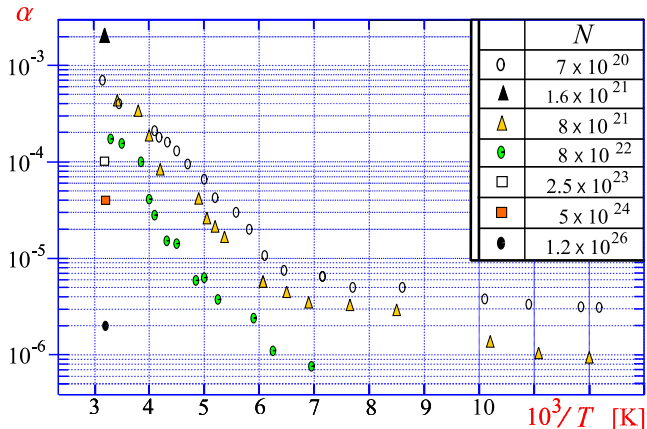


FIG. 1: Hooge's coefficient  $\alpha$  vs  $10^3/T$ . The values with  $N = 7 \times 10^{20}$ ;  $8 \times 10^{21}$ ;  $8 \times 10^{22}$  are taken from Ref. 5. The other four values, at only  $T = 300$  K, have been given us by Vandamme, and are the experimental results whence  $\alpha_{latt}$  has been derived in Ref. 6.

[i.e., the ZPF of stochastic electrodynamics (SED)] brings about a small interval  $\delta v$ , starting from a speed  $v_1$ , where the two collision frequencies  $\nu_1$  and  $\nu_2$  appearing in the Fokker-Planck equation accounting for electron-electron ( $e-e$ ) interactions [denoted as  $e-e$  FP and given by Eqs. (16) and (17) of Ref. 13] are both proportional to  $1/v$  [as given by Eq. (6) of Ref. 13], which corresponds to the threshold of runaways. The conditional (or transition) probability density in the mentioned  $\delta v$  interval turns out to be given by Eq. (30) of Ref. 13, showing an extremely slow time decay, such that the memory of a fluctuation is practically infinite. It is just starting from Eq. (30) of Ref. 13 that, in Sec. II, we derive the consequent power spectral density  $S_\varepsilon(f)$  which turns out to be

$$S_\varepsilon(f)G^{-2} = \alpha_\varepsilon(T=0, N)(2\pi\tau_m)^\varepsilon \mathcal{N}^{-1} f^{\varepsilon-1}, \quad (2)$$

where  $\tau_m$  denotes the information transmission time given by Eq. (27) of Ref. 13. Now  $\alpha_\varepsilon$  shows a never noticed dependence on the number density  $N$ , and interpolates the experimental data of Fig. 1 (we repeat: for the purest, quietest semiconductors). Another advantage of Eq. (2) is that the total noise power, i.e.,  $\int_0^{+\infty} df S_\varepsilon(f)$ , does not diverge for  $f \rightarrow 0$  because  $\varepsilon > 0$  (although it is very small). On the contrary, an exact  $1/f$  noise as expressed by Eq. (1) presents such unphysical divergence. The other unphysical divergence for  $f \rightarrow \infty$  is eliminated for both Eqs. (1) and (2) because the high speed electrons undergo inelastic scatterings with the lattice.

What we find in Sec. II (in particular, that the  $\alpha_\varepsilon$  appearing in Eq. (2) interpolates all the known experimental data for the purest semiconductors) regards an indefinite medium. Taking into account that our mechanism is not due to electron-lattice scatterings but to electron-electron interactions, it would seem at first glance that the finite transit time  $\tau_{tr}$  in a finite sample would imply

a lower cut-off at  $f_{min} \simeq 1/\tau_{tr}$ . However, it will be shown in Sec. III that the diffusion in the configuration space (for the only electrons in the small effective  $\delta v$  range) becomes ballistic, hence much larger than the drift velocity for the electrons in the same  $\delta v$  range. Consequently, the back diffusion, much larger than the drift, transmits and preserves the information of a fluctuation between the electrodes of a finite sample. That is possible also because the transmission of information is mainly due to  $e-e$  interactions and, in the effective  $\delta v$  interval, the  $e-e$  collision frequency is much larger than that between electrons and lattice.

We conclude in Sec. IV underscoring why this long-standing problem for the purest, quietest semiconductors, has required such a long time for its solution.

## II. $1/f$ IN AN INDEFINITE SEMICONDUCTOR

The adjective ‘‘indefinite’’ means that here we do not take into account the finite transient time between the electrodes. Actually, we consider a semiconductor sample having length  $L$  between the two electrodes connected to the measuring instrument, in which a uniform current density  $j$  flows through a constant cross-section  $S$  under the action of a uniform electric field  $\mathbf{E}$ . The total current  $I$  flowing in the considered sample may be expressed as

$$I = jS = eNwS = \frac{e}{L}\mathcal{N}\mu_m E = GEL, \quad (3)$$

where  $e$  is the electron charge,  $N$  the number density of the free electrons,  $\mathcal{N} = NSL$  their total number in the considered sample,  $w = \mu_m E$  the drift velocity,  $G$  the conductance, and  $\mu_m$  the mobility given by [14, 15]

$$\mu_m = \frac{e}{m_*} \langle \mu(v) \rangle_{p_0} = \frac{e}{m_*} \left\langle \frac{1}{\nu_2(v)} - \frac{v}{3\nu_2^2(v)} \frac{d\nu_2}{dv} \right\rangle_{p_0}, \quad (4)$$

$v$  being the electron speed,  $m_*$  the electron effective mass in the considered semiconductor, and  $\nu_2(v)$  the electron collision frequency. The conductance, in terms of microscopic quantities, is easily derived from Eqs. (3) and (4)

$$G(t) = \frac{e}{L^2} \mathcal{N} \mu_m = \frac{e^2}{m_* L^2} \mathcal{N} \langle \mu(v) \rangle_{p_0}, \quad (5)$$

and fluctuates in time  $t$  because the distribution function  $p_0(v, t)$  (contained in the average over  $v$ ) fluctuates in  $t$ . If we average over  $t$  or, since the process is ergodic, we take the ensemble average, we have  $\langle G(t) \rangle = G$ . The correlation function  $C_G(\tau)$  of  $G(t)$  is the same of  $g(t) = G(t) - G$ , having zero mean value. It is

$$\begin{aligned} C_G(\tau) &= \langle g(t)g(t+\tau) \rangle = \langle G(t)G(t+\tau) - G^2 \rangle \\ &= \mathcal{N} \left( \frac{e^2}{m_* L^2} \right)^2 \int_0^\infty dv_0 4\pi v_0^2 p_0(v_0) \mu(v_0) \\ &\quad \times \int_0^\infty dv 4\pi v^2 [p_0(v, \tau|v_0) - p_0(v)] \mu(v), \end{aligned} \quad (6)$$

where  $p_0(v, \tau|v_0)$  is the transition probability density (or Green's solution) to have the speed  $v$  at time  $\tau$  beginning from  $v_0$  at  $\tau = 0$  and  $p_0(v) = p_0(v, \infty|v_0)$ . As said, it has been shown in Ref. 13 that the real ZPF of SED always brings about a  $\delta v$  interval, starting from  $v_1$ , where the two collision frequencies  $\nu_1$  and  $\nu_2$  appearing in the  $e - e$  FP become  $\nu_1 \propto \nu_2 \propto 1/v$ , corresponding to the threshold of runaways. In that effective  $\delta v$  interval,  $p_0(v, \tau|v_0) - p_0(v)$  decays, in an extremely slow way, as  $\tau^{-\varepsilon}$  with  $0.003 \leq \varepsilon \leq 0.007$ . Consequently, also the correlation function (6) decays in the same way. Since the effective  $\delta v$  is unique, it is therefore convenient to split the second integral of Eq. (6) in three parts, the first from 0 to  $v_1$ , the second from  $v_1$  to  $v_1 + \delta v$ , and the third from  $v_1 + \delta v$  to  $\infty$ . In the first part there are two small  $v$  intervals in one of them  $\nu_1 \propto 1/v$ , and in the second  $\nu_2 \propto 1/v$ . Being the two  $v$  intervals different, according to Fig. 1 of Ref. 13,  $F(\tau) = p_0(v, \tau|v) - p_0(v)$  decays as a mixture of exponentials and powers of  $\tau$ . The consequent power spectral noise is a mixture of  $f^{-n}$  with  $n \geq 2$  and white noises. In the third part there is no  $v$  interval where at least one  $\nu \propto 1/v$ , so that the consequent noise is a sum of Lorentzian, i.e., of white noises. The second part from  $v_1$  to  $v_1 + \delta v$  is the only one useful to produce  $1/f^{1-\varepsilon}$  noise. In the effective  $\delta v$  interval, the transition probability density is given by Eq. (30) of Ref. 13, which vanishes for  $\tau \rightarrow \infty$ , so that  $p_0(v) = 0$ . The contribution of the second part is therefore

$$C_G^{\delta v}(\tau) = \left( \frac{e^2}{m_* L^2} \right)^2 \mathcal{N} \int_0^\infty dv_0 4\pi v_0^2 p_0(v_0) \mu(v_0) \times \int_{v_1}^{v_1 + \delta v} dv p_0(v_1) \frac{4\pi}{v_1^{1-\varepsilon}} \frac{v_1^{3-\varepsilon} \tau_m^\varepsilon}{(\tau + \tau_m)^\varepsilon} \mu(v). \quad (7)$$

Again for  $v_1 \leq v \leq v_1 + \delta v$ , we derive from Eq. (6) of Ref. 13 and from Eq. (4)

$$\mu(v) = \frac{4}{3} v (BK)^{-1}. \quad (8)$$

Consequently, the second integral of Eq. (7) can easily be performed and does not depend on  $v_0$ . The first integral is therefore  $\langle \mu(v) \rangle_{p_0}$ , so that Eq. (7) reduces to

$$C_G^{\delta v}(\tau) = \left( \frac{e^2}{m_* L^2} \right)^2 \mathcal{N} \langle \mu(v) \rangle_{p_0} \frac{16\pi p_0(v_1) v_1^3 \tau_m \delta v}{3BK(\tau + \tau_m)^\varepsilon}. \quad (9)$$

The power spectral density of  $G(t)$  is the Wiener-Khinchine transform of its correlation

$$S(f) = 2 \int_0^\infty d\tau C_G^{\delta v}(\tau) \cos(2\pi f\tau). \quad (10)$$

To the aim of only finding  $1/f^\varepsilon$ , we derive from Eqs. (5), (8)-(10)

$$\frac{S_f}{G^2} = \frac{32\pi p_0(v_1) v_1^3 \delta v \tau_m^\varepsilon}{\mathcal{N} 3BK \langle \mu(v) \rangle_{p_0}} \int_0^\infty \frac{d\tau}{(\tau + \tau_m)^\varepsilon} \cos(2\pi f\tau). \quad (11)$$

$\frac{N}{\text{m}^{-3}}$	$B$	$\frac{K}{\text{ms}^{-2}}$	$\frac{\langle \mu(v) \rangle_{p_0}}{\text{m}^3 \text{s}^{-2}}$	$\alpha_\varepsilon$
$10^{20}$	0.172	$1.37 \times 10^{19}$	$3.97 \times 10^4$	$2.64 \times 10^{-5}$
$10^{21}$	0.112	$3.58 \times 10^{20}$	$2.71 \times 10^4$	$1.96 \times 10^{-6}$
$10^{22}$	0.113	$9.01 \times 10^{21}$	$1.15 \times 10^4$	$1.89 \times 10^{-7}$
$10^{23}$	0.115	$2.07 \times 10^{23}$	$3.18 \times 10^3$	$3.02 \times 10^{-8}$
$10^{24}$	0.191	$4.11 \times 10^{24}$	$3.25 \times 10^2$	$7.15 \times 10^{-9}$
$10^{25}$	0.236	$6.16 \times 10^{25}$	$0.42 \times 10^2$	$2.32 \times 10^{-9}$
$10^{26}$	0.248	$6.11 \times 10^{26}$	$0.45 \times 10^1$	$1.55 \times 10^{-9}$

TABLE I: Values of the fundamental parameters vs the number density  $N$  of free electrons.

Setting  $2\pi f\tau = x$  and  $2\pi f\tau_m = x_m$ , the integral of Eq. (11) becomes

$$\frac{1}{(2\pi f)^{1-\varepsilon}} \int_0^{+\infty} dx \frac{\cos x}{(x_m + x)^\varepsilon} = \frac{\varepsilon(\pi/2)}{(2\pi f)^{1-\varepsilon}}, \quad (12)$$

showing that the power spectral density of the noise is actually of the kind  $1/f^{1-\varepsilon}$ . The integral in  $x$  has been calculated after performing an integration by parts taking  $dx \cos x$  as the differential factor.

Substituting Eq. (12) into Eq. (11) we obtain

$$\frac{S_f}{G^2} = \frac{8\pi v_1^3 p_0(v_1) \varepsilon \delta v (2\pi \tau_m)^\varepsilon}{\mathcal{N} 3 \langle \mu(v) \rangle_{p_0} BK f^{1-\varepsilon}}. \quad (13)$$

Comparing Eq. (13) with Eq. (2) we obtain

$$\alpha_\varepsilon = \alpha_\varepsilon(T=0, N) = \frac{8\pi v_1^3 p_0(v_1) \varepsilon \delta v}{3 \langle \mu(v) \rangle_{p_0} BK}. \quad (14)$$

The values of the quantities appearing in Eq. (14) are partially reported in Table 1 of the companion paper [13], and the rest in Table I. Since the collision frequency  $\nu_2(v)$  appearing in Eq. (4) and in the  $e-e$  FP has been obtained numerically by means of Eq. (39) of our previous paper [15], it has been convenient to perform an integration by parts of Eq. (4), thus obtaining

$$\langle \mu(v) \rangle_p = -\frac{4}{3} \pi \int_0^{+\infty} dv \frac{v^3}{\nu_2(v)} \frac{\partial p_0(v)}{\partial v}, \quad (15)$$

which is a standard expression[14, 15] containing the derivative of  $p_0(v)$  analytically given by Eq. (18) of Ref. 13.

In Table I of Ref. 13, and in Table I of the present paper, there are the two fundamental results, namely, i) the exponent  $\varepsilon$  of the time  $\tau$  decay of the effective part of the correlation function (12), which leads to  $f^{\varepsilon-1}$  of Eq. (13); ii) the parameter  $\alpha_\varepsilon$  of the Hooke-like coefficient which, differently from Hooke's, depends on  $N$ . With the average value  $\varepsilon = 0.005$ , and Eq. (27) of Ref. 13, we obtain  $(2\pi \tau_m)^\varepsilon = 0.96$  [s<sup>0.005</sup>], which gives a pure number with  $f^\varepsilon$ . The  $\delta v$  values are obtained taking  $v_1$  and  $v_1 + \delta v$  as the  $v$  values at which  $\nu_2(v)v$  decreases by 0.5% with respect to the maximum value  $[\nu_2(v)v]_M$  of the plateau. Actually, because of the inaccuracy of the calculated data, we almost do not appreciate any variation

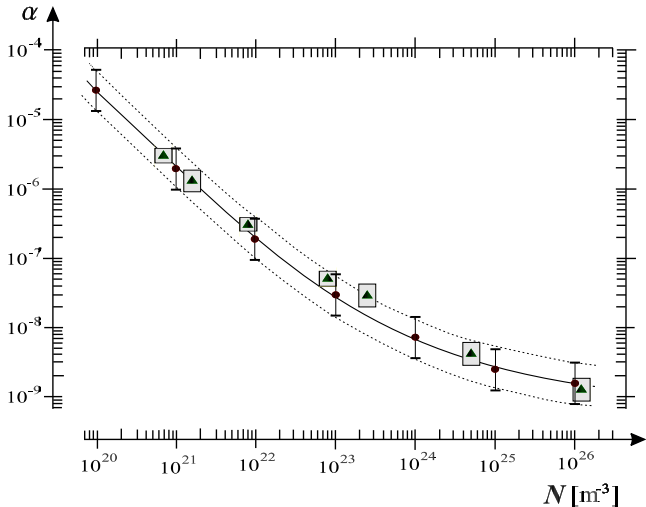


FIG. 2: Plot of  $\alpha_\varepsilon$  vs the number density  $N$ . The solid line interpolates the calculated values (triangles). The experimental data  $\alpha_{\text{exp}}$  are denoted by black circles with error bars.

of  $\nu_2(v)v$  to within 0.5% of the peak value. In any case, both the approximations in the calculated values, and the criterion of delimiting the effective  $\delta v$  interval, are the main cause of errors, we have estimated to be  $\pm 40\%$ , even after taking the line interpolating the calculated results vs  $N$ . The second cause of error is the evaluation of  $\varepsilon$  that is expressed, in Eq. (21) of Ref. 13, as the difference between 3 and  $3BK^2/a^2$ , the latter requiring an accuracy of 4 significant figures in order to ensure a single figure for  $\varepsilon \simeq 0.005$ . That is why only one significant figure appears in Table 1 of Ref. 13. This implies an error  $\pm 15\%$  for the maximum  $\varepsilon$  value (0.007), and an error  $\pm 30\%$  for the minimum  $\varepsilon$  value (0.003). Summing quadratically the errors, the total uncertainty is between 0.43 and 0.5. The results for different number densities  $N$  are reported in Table I and can be summarized by

$$\alpha_\varepsilon = \alpha_{\varepsilon 0}(N_0) \left( \frac{N_0}{N} \right)^{0.898 - 0.092 \log(N/N_0)} (1 \pm 0.45), \quad (16)$$

where  $N_0 = 10^{22} \text{ m}^{-3}$ , and  $\alpha_{\varepsilon 0}(N_0) = 1.89 \times 10^{-7}$ .

If we compare Eq. (13) with the normally used Eq. (1) [instead of with Eq. (2)], we obtain

$$\alpha(T=0, N) = \alpha_\varepsilon(T=0, N) (2\pi\tau_m f)^\varepsilon. \quad (17)$$

Fortunately the dependence on  $f$  is so small ( $\varepsilon \simeq 0.005$ ) that even for the minimum  $f_{\text{min}} = 2.8 \times 10^{-7} \text{ s}^{-1}$  (corresponding to 40 days), and  $\tau_m$  given by Eq. (27) of Ref. 13, it is  $(2\pi\tau_m f_{\text{min}})^\varepsilon \simeq 0.89$ . For  $f_{\text{max}} \simeq 10^4 \text{ s}^{-1}$  (where the thermal noise becomes relevant), it is  $(2\pi\tau_m f_{\text{max}})^\varepsilon \simeq 1.01$ . For the minimum ( $\simeq 1 \text{ Hz}$ ) and maximum ( $\simeq 10^4 \text{ Hz}$ )  $f$  values used in the experiments whose results are reported in Fig. 1, it is  $0.96 < (2\pi\tau_m f)^\varepsilon < 1.01$ , so that, taking the average value, we have  $\alpha(T=0, N) \simeq 0.98\alpha_\varepsilon(T=0, N)$ . We see that the relative differences are much smaller than the

other theoretical uncertainties. For simplicity we report  $\alpha_\varepsilon$  in Fig. 2, because it is theoretically independent on  $f$ . What is important is that our  $\alpha_\varepsilon$  must not be compared with Hooge's [3]  $\alpha_{\text{latt}}$ , because the present theory shows that pure  $1/f$  noise is almost exclusively given by electron-electron interactions (thus explaining its universality for the conduction current, independently of the material), and therefore it has nothing to do with a presumed interaction with the lattice. The comparison has to be done directly with the experimental results relevant to pure semiconductors (which are also the quietest ones). The only data clearly extrapolable to  $T=0$  are those of Ren and Hooge [5]. The interpolation of the raw data leads to an expression similar to the one of Hooge [3] for his  $\alpha_{\text{latt}}$ , although we have also here found an  $N$  dependence

$$\alpha = \alpha_0(T=0, N) + b(N) \exp \left[ -\frac{\Delta E(N)}{kT} \right]. \quad (18)$$

Since  $b > 10^3 \alpha_0$ , for the highest  $\alpha$  values (corresponding to the highest used temperatures  $T$ ), it is easy to find  $b(N)$  and  $\Delta E(N)$ . The latter slightly depends on  $N$  and it is therefore easily extrapolable. At this point we can also exploit the measurements of Hooge and Vandamme [6], whose raw data have been kindly given us directly by Vandamme. If we plot them vs  $N$  we find  $b(N) = b(N_1)(N_1/N)^{0.43}$ , with an uncertainty  $\simeq 65\%$  for the value at the maximum experimented  $N$  value ( $N = 1.2 \times 10^{26} \text{ m}^{-3}$ ), where the measurements of the very small noise is difficult. If we also include the Ren-Hooge [5] data, we improve the accuracy of  $b(N_1)$  which turns out to be  $b(N_1) = 2 \times 10^{-3}$  for  $N_1 = 1.6 \times 10^{21} \text{ m}^{-3}$ . By means of this more accurate  $b(N)$ , we can now obtain more reliable  $\alpha_0(T=0, N)$  values from the curves interpolating the Ren-Hooge data [5]. We get  $\alpha_0(N = 7 \times 10^{20} \text{ m}^{-3}) = 3 \times 10^{-6}$ ;  $\alpha_0(N = 8 \times 10^{21} \text{ m}^{-3}) = 3 \times 10^{-7}$ ;  $\alpha_0(N = 8 \times 10^{22} \text{ m}^{-3}) = 5 \times 10^{-8}$ . The estimated error is  $\simeq 15\%$ , while for the values extrapolated from the Hooge-Vandamme [6] results is  $\simeq 30\%$ . We have obtained for the latter ones:  $\alpha_0(N = 1.6 \times 10^{21} \text{ m}^{-3}) = 1.3 \times 10^{-6}$ ;  $\alpha_0(N = 2.5 \times 10^{23} \text{ m}^{-3}) = 2.85 \times 10^{-8}$ ;  $\alpha_0(N = 5 \times 10^{24} \text{ m}^{-3}) = 4.1 \times 10^{-9}$ ;  $\alpha_0(N = 1.2 \times 10^{26} \text{ m}^{-3}) = 1.2 \times 10^{-9}$ . The first extrapolated values corresponds to a number density inside the range delimited by the Ren-Hooge results, and the second value is just outside that range. Their differences from the line interpolating the Ren-Hooge values are well within the estimated experimental uncertainty. We can therefore rely on the last two extrapolated values. All the seven values of the  $\alpha_0$  with their uncertainties are reported in Fig. 2 vs  $N$ , together with our theoretical values with their band of uncertainty. We see that the agreement is well inside the uncertainties, and that the line interpolating the values derived from the experimental raw data is in excellent agreement with the line expressed by Eq. (16). We again emphasize that the  $N$  dependence, never considered before, sets in order the apparent great dispersion of the experimental data.

The preceding ideas and results are sufficient to explain in a satisfactory way the  $1/f^{0.995}$  noise in an indefinite medium. The point is that the electrons between the two electrodes (where a fluctuating voltage is measured)  $L$  apart from each other, take a time interval  $L/w$  (where  $w$  is the drift velocity, only due to  $\mathbf{a}_{D.C.}$ ) to traverse  $L$ . But there is no evidence of a tiny change in the decay after  $L/w$ . The solution is due to the rapid back diffusion for the fraction of the electrons whose speed is inside the small, effective time interval  $\delta v$ . The diffusion velocity (for only that small fraction) turns out to be much larger than the drift velocity. More in detail, the explanation is based on the following two results: i) The electron-electron ( $e-e$ ) scattering is dominant for the generation of  $1/f^{1-\varepsilon}$  noise. ii) The  $e-e$  scattering also preserves the memory of a fluctuation much beyond the average transit-time  $L/w$ . In fact, free electrons are subjected to the drift velocity due to  $\mathbf{a}_{D.C.}$ , but also to diffusion due to  $\mathbf{a}_{ZPF}$ . Now, during the time  $\tau_m$  of information transmission given by Eq. (27) of Ref. 13, the average displacement due to the drift velocity is

$$\delta x = w\tau_m = \mu_m E_{D.C.} \tau_m. \quad (19)$$

The most probable displacement in the same time interval, due to the longitudinal diffusion coefficient  $D_L$  for the electrons in the useful  $\delta v$  range, is [15, 16]

$$\delta r = [4D_L(v_1)\tau_m]^{1/2}, \quad (20)$$

where  $v_1$  is the smallest velocity of the useful  $\delta v$  range. Now, as shown by Parker and Lowke [16], when  $\nu \propto v^{-1}$ , i.e., at the threshold of runaways,  $D_L$  diverges. In fact, the expression of  $\nu$  considered after Eq. (13) of Ref. 16 (p.293), is  $\nu = \nu_0(\epsilon/\epsilon_0)^{(l+1)/2} = \nu_0(v/v_0)^{l+1}$  (since  $\epsilon = mv^2/2$ ) and they found

$$D_L/D_T = (l+3)/[2(l+2)], \quad (21)$$

where  $D_T$  is the transversal diffusion coefficient. We see clearly that this ratio diverges for  $l \rightarrow -2$ , corresponding to  $\nu \propto v^{-1}$ . Parker and Lowke [16] obtained it by a semi-quantitative model (their Sec. III), and their quantitative theory implies a still stronger divergence, as can be seen comparing this ratio with Table I of Ref. 16. Since  $D_L$  is defined as  $\lim_{t \rightarrow \infty} \langle (x-x_0)^2/t \rangle$ , the divergence means that  $(x-x_0)^2 \propto t^2$ , i.e., the diffusion becomes ballistic and  $\delta r \simeq v_1\tau_m$ . The ratio between  $\delta r$  and the  $\delta x$  given by Eq. (19), i.e.,  $\delta r/\delta x = v_1/(\mu_m E_{D.C.})$ , is very large [17], and independent of  $\tau_m$ . Consequently, the velocity of back diffusion, responsible for the transmission of information to the new electrons entering the  $L$  section of the sample (between the two electrodes  $L$  apart from each other), is much larger than the drift velocity  $w$ . That is why the memory of a fluctuation is preserved independently of the transit-time  $L/w$ . It was just the divergence of  $D_L$  when  $\nu \propto v^{-1}$  that suggested to one

of us (G. Cavalleri) the idea of the possible origin of  $1/f$  noise when  $\nu_2 \propto v^{-1}$ , because of the connection between noise power spectral densities and generalized diffusion coefficients [18]. The pure  $1/f$  noise is therefore valid for both an indefinite medium and a small sample.

#### IV. CONCLUSIONS

When a fluctuation produces a pimple in the distribution function of the electron speeds, the pimple tends to diffuse and drift in the speed space because of collisions. However, at the threshold of runaways there is a kind of counter-diffusion and counter-drift in the speed space, so that the pimple appears as almost crystallized, decaying as  $\tau^{-0.005}$ . Moreover, this result is independent of the transit time  $L/w$  of the electrons. What is more, our theoretical expression (16) fits the experimental data much better than Hooge's empirical formula, because we find a dependence on the electron concentration  $N$  besides the total number  $\mathcal{N}$  of electrons in the considered sample. No previous paper has ever predicted the  $N$  dependence that is peculiar for the pure  $1/f$  noise, and not for the  $1/f$  like component due to electron trapping-detrappings. The data fittings hold only for the pure  $1/f$  noise, i.e., the one present in the purest and quietest semiconductors. The pure  $1/f$  noise can also be obtained as the residue of the usual much larger  $1/f$  like noise after subtracting the usually much larger contribution due to trapping-detrappings.

Finally, the  $1/f^{1-\varepsilon}$  noise only depends on the electron-electron ( $e-e$ ) scattering in a small  $\delta v$  range, and it is therefore independent of the electron-lattice ( $e$ -lattice) scattering. The coefficient of proportionality  $\alpha_\varepsilon$  depends on the material only via the mobility  $\langle \mu(v) \rangle_{p_0}$  appearing at the denominator of Eq. (14). In turn,  $\langle \mu(v) \rangle_{p_0}$ , averaged over  $0 < v < \infty$ , is due to both  $e-e$  and  $e$ -lattice scatterings ( $e-e$  scattering is much larger than  $e$ -lattice scattering only in the neighborhoods of the effective  $\delta v$  interval).

The pure  $1/f$  noise is therefore fully explained from both the experimental and theoretical points of view. The reason why this long standing problem challenged all the previous attempts is that it required a catena of successive achievements, we summarize below: 1) The reduction of the nonlinear Boltzmann equation with electron-electron ( $e-e$ ) interaction to a Fokker-Planck (FP) equation; 2) The steady-state solution  $p_0(v)$  of  $e-e$  FP equation, which depends on the square of acceleration  $\mathbf{a}$ ; 3)  $p_0(v)$  becomes similar to the Fermi-Dirac distribution function if  $a^2$  is caused by the zero-point field (ZPF) of QED. It is just because of  $a_{ZPF}^2$  that there is a small interval  $\delta v$  for the electron speed  $v$  where runaways occur; 4) In this  $\delta v$  range, the time-dependent Green's solution of the  $e-e$  FP decreases as  $\tau^{-\varepsilon}$  with  $\varepsilon \leq 0.007$ . Then,  $S(f) \propto 1/f^{1-\varepsilon}$  and also depends on the electron concentration, thus closely fitting the experimental data; 5) In a finite sample, fluctuations are remembered because

back diffusion is much more rapid than drift velocity.

Indirectly, the qualitative and quantitative explanation of the universal pure  $1/f$  noise is a new proof of the

existence of the real (i.e., unrenormalized) zero-point field (ZPF) of SED.

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  - [17] Notice that the rapid diffusion is practically not detectable experimentally, because it is relevant to a number  $\delta\mathcal{N} \simeq 10^{-3}\mathcal{N}$  of electrons in the small speed interval  $\delta v \simeq 2 \times 10^{-3}\langle v^2 \rangle^{1/2}$ . Moreover, the diffusion coefficient (for only the electrons in  $\delta v$ ) tends to diverge in the configuration space, while tends to vanish in the speed space.
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