Expanding or Non-Expanding Universe

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Abstract: The observed redshift of galaxies in the universe is generally interpreted as expansion of space. In a previous paper the author has given an other interpretation. It has been proved that the proper time is not absolute but the so called observer's time corresponding to the proper time at present is absolute. This gives the observed redshift of galaxies. In this preprint several results of an expanding and a non-expanding universe are compared with one another. There exists no definite answer whether the universe expands or not. Einstein's theory suggests an expanding space and flat space-time theory of gravitation suggests a non-expanding space.

1. Introduction

The redshift of distant galaxies in the universe is generally interpreted as expansion of space. Homogeneous, isotropic, cosmological models of Einstein's general theory of relativity suggest this interpretation. Hubble had doubts about the reality of the expansion but Sandage and collaborators believe that the space is really expanding. For a discussion of these two interpretations of the redshift of distant galaxies compare the paper of Soares [1] where further references can be found. The interpretation of the redshift as expansion of space is generally accepted but some problems with this interpretation exist (see, e.g. [2-4]).

In addition to Einstein's general theory of relativity a covariant theory of gravitation in flat space-time will be considered (see Petry [5]). This gravitational theory is based on a flat space-time metric and gravitation is described by a field in analogy to Maxwell's theory. It gives the same results as Einstein's general theory of relativity to the experimentally needed accuracy for the following effects: gravitational redshift, light deflection, perihelion precession, radar time-delay, post-Newtonian approximation, gravitational radiation and the precession of the spin axis of a gyroscope in the orbit of a rotating body. The theory of gravitation in flat space-time gives non-singular, homogeneous, isotropic, cosmological models in contrast to Einstein's theory (see, e.g. Petry [6-11]). The space of the universe in flat space-time theory of gravitation is flat for all cosmological models as recent observations also suggest. In addition to the proper time (atomic time) in the universe the observer's time is introduced (see Petry [12]) which corresponds to the proper time at present. The observer's time interval and the proper time interval are different and they agree only at present. This result has been used to give an explanation of the anomalous Doppler frequency shift of the Pioneers [12] and an explanation of the redshift of galaxies without the assumption of an expanding space (see [13]). It is proved that the observer's time interval is absolute [13] suggesting that the time interval of proper time is changeable in the universe. An Euclidean space and different times such as observer's time resp. proper time are used to study non-expanding cosmological models. Cosmological models with an expanding space and the use of the proper time are also considered. The light velocity in the universe depends on the used time. It is equal to the vacuum light velocity for all times only by the use of observer's time. It is using an other time interval only at the observer equal to the vacuum light velocity whereas at distant objects it is different from it and it depends in an expanding space also on the expansion of space as well for flat space-time theory of gravitation as for Einstein's general theory of relativity. Therefore, superluminal velocities in an expanding space are real and not contradicting to special relativity. The energy of an emitted photon moving in the universe is in general time-depenent and only by the use of the observer's time it is always constant because the observer's time is absolute. The total energy of the universe is conserved and positive for all cosmological models (including an expanding space) by flat space-time theory of gravitation. This follows from the fact that the total energy-momentum in flat space-time theory of gravitation is a tensor. For a non-expanding flat space of Einstein's theory the total energy of the universe is equal to zero (see [14]). It is worth mentioning that a total energy of the universe being equal to zero in flat space-time theory of gravitation would also give singular cosmological models like Einstein's theory. In flat space-time theory of gravitation a bound system in the universe is calculated to post-Newtonian approximation [15]. The result to Newtonian accuracy is identical with that of general relativity. The simple case of a solid, spherically symmetric body and a test particle moving around it is studied. The results of an expanding or a non-expanding space mathematically agree but the interpretation of an expanding universe yields that the space of the bound system does not expand [16] whereas the interpretation of a non-expanding space implies that the test body moves spirally towards the solid body.

It is worth mentioning that the two considered theories of gravitation allow the interpretation of an expanding and a non-expanding space for all the studied results but the interpretation of expanding space is suggested by Einstein's theory whereas flat space-time theory of gravitation suggests a non-expanding space. This follows from the fact that Einstein's theory defines the proper time and the metric of space and time by the same line-element whereas flat space-time theory of gravitation defines proper time indepenently of the background metric of space and time.

2. Summary of some Results

The theory of gravitation in flat space-time [5] has been studied in several papers. A summary of the theory with applications can be found in paper [6] where references to detailed studies are stated.Subsequently, some results of the papers [5-11] are summarized which are used in the following.

Flat space-time theory of gravitation uses a flat space-time background metric

$$(ds)^2 = -\eta_{ij} dx^i dx^j . \tag{2.1}$$

The gravitational field is described by a symmetric tensor g_{ij} satisfying covariant (with regard to the flat space-time metric (2.1)) differential equations of order two. The source of the gravitational field is the total energy-momentum tensor inclusive that of the gravitational field. The proper time (atomic time) is defined by

$$c^2 (d\tau)^2 = -g_{ij} dx^i dx^j . \tag{2.2}$$

The equations of motion of a test particle with the four-velocity $\left(\frac{dx^i}{d\tau}\right)$ in the gravitational field are given by

$$\frac{d}{d\tau}\left(g_{ij} \frac{dx^j}{d\tau} \right) = \frac{1}{2} \frac{\partial g_{jk}}{\partial x^i} \frac{dx^j}{d\tau} \frac{dx^k}{d\tau} \quad (i = 1-4.) \tag{2.3}$$

The total energy-momentum tensor is conserved. The derivation of these results can be found in the paper of Petry [5] . The application of the theory of gravitation in flat space-time to homogeneous, isotropic, cosmological models is studied in several papers (see, e.g. Petry [6-10]). The background metric uses the pseudo-Euclidean geometry

$$(\eta_{ij}) = diag(1,1,1,-1) \tag{2.4}$$

with the Cartesian coordinates x^1, x^2, x^3 and $x^4 = ct$. The four-velocity of the universe is

$$u^i = \frac{dx^i}{d\tau} = 0 \quad (i = 1,2,3) \;, \qquad u^4 = c \frac{dt}{d\tau} . \tag{2.5}$$

Then, the potentials g_{ij} are given by

$$(g_{ij}) = diag(a^2(t), a^2(t), a^2(t), -1/h(t)) \tag{2.6}$$

where $a(t)$ and $h(t)$ satisfy two coupled nonlinear differential equations of order two resulting from the field equations of the gravitational theory. Here, the source of the gravitational field consists of the densities of matter (dust) $\rho_m(t),$ of radiation $\rho_r(t),$ and of the cosmological constant $\rho_\Lambda(t)$ with

$$\rho_m(t) = \rho_{m0} / \sqrt{h(t)},\; \rho_r(t) = \rho_{r0} / \left(a(t)\sqrt{h(t)}\right),\; \rho_\Lambda(t) = \frac{\Lambda c^2}{8\pi k} a^3(t) / \sqrt{h(t)}. \tag{2.7}$$

The parameters ρ_{m0} and ρ_{r0} are the densities at present, Λ is the cosmological constant and k is the gravitational constant.

The initial conditions of the differential equations are the values at present time $t = 0,$ i.e.

$$a(0) = h(0) = 1, \quad \dot{a}(0) = H_0, \quad \dot{h}(0) = \dot{h}_0 \tag{2.8}$$

where the dot denotes the $t-$derivative. Here, H_0 denotes the Hubble constant and $\dot{h}_0$ is a further constant of integration which is zero for Einstein's theory because $h(t) \equiv 1$ which is not possible in flat space-time theory of gravitation. Under natural conditions on the universe the cosmological models are nonsingular, i.e. all the densities are finite for all times. The functions $a(t)$ and $h(t)$ are defined for all $t \in$ R. The function $h(t)$ goes to infinity as $t \to -\infty,$ then for increasing time it decreases to a small positive minimum and then increases to infinity as $t \to \infty.$ The function $a(t)$ starts from a small positive value as $t \to -\infty,$ then for increasing time it decreases to a positive minimum and then increases for all $t \in$ R. The functions $a(t)$ and $h(t)$ have their minimum in the very early universe. The energy density of the gravitational field in the universe is

$$\rho_G(t)=\frac{1}{64\pi k}a^3\sqrt{h}\left(-6\left(\frac{\dot{a}}{a}\right)^2+6\frac{\dot{a}}{a}\frac{\dot{h}}{h}+\frac{1}{2}\left(\frac{\dot{h}}{h}\right)^2\right). \tag{2.9}$$

The space of the universe is flat which is suggested by recent observations.The energy-momentum tensor of the total universe is given by

$$\begin{aligned} T^1{}_1 = T^2{}_2 = T^3{}_3 &= \frac{1}{3}\rho_r c^2 - \rho_\Lambda c^2 + \rho_G c^2 \\ T^4{}_4 &= -(\rho_m + \rho_r)c^2 - \rho_\Lambda c^2 - \rho_G c^2 \end{aligned} \tag{2.10}$$

where the non-diagonal elements are equal to zero. The total energy of the universe is conserved, i.e.

$$(\rho_m(t)+\rho_r(t)+\rho_\Lambda(t)+\rho_G(t))c^2 = \lambda c^2 \tag{2.11}$$

with a constant λ >0. Einstein's theory gives for a flat space that the total energy is zero (see, e.g.Berman [14]). In addition to the time t the proper time $\tilde{t}$ of the observer at rest is

$$d\tilde{t} = dt/\sqrt{h(t)} \tag{2.12a}$$

with

$$\tilde{t}(t) = \int_{-\infty}^{t} dt/\sqrt{h(t)} \tag{2.12b}$$

implying the potentials

$$(g_{ij}(\tilde{t})) = diag(a^2(t), a^2(t), a^2(t), -1) \tag{2.13}$$

and the background metric

$$(\eta_{ij}(\tilde{t})) = diag(1,1,1,-h(t)). \tag{2.14}$$

In the relations (2.13) and (2.14) the time t must be replaced by $\tilde{t}$ by the use of inverse relation of (2.12b). Furthermore, let us define the observer's time t' by

$$dt' = dt/\left(a(t)\sqrt{h(t)}\right) \tag{2.15a}$$

with

$$t'(t) = \int_{-\infty}^{t} dt/\left(a(t)\sqrt{h(t)}\right) \tag{2.15b}$$

implying the potentials

$$(g_{ij}(t')) = diag(a^2(t), a^2(t), a^2(t), -a^2(t)) \tag{2.16}$$

and the flat background metric

$$(\eta_{ij}(t')) = diag(1,1,1,-a^2(t)h(t)) \tag{2.17}$$

where relation (2.15b) must be used to replace t by t'. The absolute values of the light velocity v_l with regard to the three different times $t, \tilde{t}$ and t' in the universe are

$$|v_l(t)| = c/\left(a(t)\sqrt{h(t)}\right), \quad |\tilde{v}_l(t)| = c/a(t), \quad |v'(t)| = c. \tag{2.18}$$

Hence, the light velocity is only with regard to the observer's time equal to c for all times whereas for t and $\tilde{t}$ the light velocity is time dependent and it is only equal to c at present by virtue of the initial conditions (2.8). It follows by the initial conditions (2.8) and the use of (2.12a) and (2.15a) at present time $t = 0$:

$$dt = d\tilde{t} = dt'. \tag{2.19}$$

Therefore, the frequency of a photon emitted at present by an atom at rest is by (2.19) independent of the used time $t, \tilde{t}$ or t'. Relation (2.1) together with the background metric (2.4) or (2.14) or (2.17) suggests that (x^1, x^2, x^3) are the Cartesian coordinates of an Euclidean non-expanding space.

Let us now consider the time t' with the relations (2.16) and (2.17). It follows from (2.2) by the use of (2.16) under the assumption that a light ray is emitted at distance r and at time t_e' resp. $t_e' + dt_e'$ and received by the observer at time $t' = 0$ resp. $0 + dt'$ then it holds

$$r = c\int_{t_e'}^{0} dt = c\int_{t_e'+dt_e'}^{0+dt'} dt = c\int_{t_e'}^{0} dt + c\left(dt' - dt_e'\right).$$

Hence,we have

$$dt' = dt_e'$$

i.e. dt' is independent of the distance r and of the time t'. Therefore, dt' is the absolute time interval and the proper time $d\tilde{t}$ is not absolute in contradiction to the general assumption. At present the time interval dt' agrees with the present atomic time interval $d\tilde{t}$ by vitue of (2.19).

Let us now introduce an expanding space by the new coordinates

$$\tilde{x}^i = a(t)x^i \quad (i = 1,2,3), \qquad d\tilde{x}^4 = dx^4 / \sqrt{h(t)}. \tag{2.20}$$

Elementary calculations give the potentials (see, e.g. [10])

$$g_{ij} = \delta_{ij}, \quad i, j = 1,2,3$$

$$= -\frac{1}{c}\frac{\dot{a}}{a}\sqrt{h}\tilde{x}^i, \qquad i = 1,2,3, j = 4$$

$$= -\frac{1}{c}\frac{\dot{a}}{a}\sqrt{h}\tilde{x}^j, \qquad i = 4, j = 1,2,3$$

$$= -\left(1 - \frac{1}{c^2}\left(\frac{\dot{a}}{a}\sqrt{h}\right)^2 \sum_{k=1}^{3}\left(\tilde{x}^k\right)^2\right), \qquad i = j = 4. \tag{2.21a}$$

The background metric has the form

$$\eta_{ij} = \frac{1}{a^2}\delta_{ij}, \quad i, j, = 1,2,3$$

$$= -\frac{1}{c}\frac{1}{a^2}\frac{\dot{a}}{a}\sqrt{h}\tilde{x}^i, \quad i = 1,2,3, j = 4$$

$$= -\frac{1}{c}\frac{1}{a^2}\frac{\dot{a}}{a}\sqrt{h}\tilde{x}^j, \quad i = 4, j = 1,2,3$$

$$= -h\left(1 - \frac{1}{c^2}\frac{1}{a^2}\left(\frac{\dot{a}}{a}\right)^2 \sum_{k=1}^{3}\left(\tilde{x}^k\right)^2\right), \qquad i = j = 4. \tag{2.21b}$$

It easily follows

$$\det\left(g_{ij}\right) = -1, \qquad \det\left(\eta_{ij}\right) = -h / a^6. \tag{2.22}$$

The radial light velocity in the expanding universe follows from (2.2) with (2.21a) implying

$$\frac{d\tilde{r}}{d\tilde{t}} = \frac{d\tilde{r}}{dt}\frac{dt}{d\tilde{t}} = \pm c + \frac{\dot{a}}{a}\sqrt{h}\tilde{r} \tag{2.23}$$

where $\tilde{r}$ denotes the Euclidean norm of $\left(\tilde{x}^1, \tilde{x}^2, \tilde{x}^3\right)$. Here, the upper (lower) sign stands for light moving away (towards) the observer. Hence, in an expanding universe the light velocity can be superluminal for sufficiently large distances. The assumption of the constant light velocity c in the expanding universe must led to contradictions. Let us now consider the expansion of space for a fixed distance vector $\left(x^1, x^2, x^3\right)$ then by equation (2.20)

$$\frac{d\tilde{x}^i}{d\tilde{t}} = \dot{a}\frac{dt}{d\tilde{t}}x^i = \frac{\dot{a}}{a}\sqrt{h}\tilde{x}^i. \tag{2.24}$$

Equation (2.2) together with (2.21a) gives by the use of (2.24)

$$\left(d\tau\right)^2 = \left(d\tilde{t}\right)^2.$$

Therefore, for any distant object in the expanding universe the introduction of the proper time $\tilde{t}$ is the natural time, i.e. any observer in the universe can use the invariant proper time.

It is worth mentioning that Einstein's general theory of relativity is working without any background metric (2.1) and relation (2.2) defines the metric in addition to the proper time. For homogenious, isotropic, cosmological models the funtion $h(t)$ is identical one. Furhtermore, there exists no covariant energy-momentum. The total energy of the universe with a flat space is identical zero. Here, the metric (2.2) with (2.13) is used, i.e. a non-expanding space is considered. The result for the total energy in the case of an expanding space, i.e. relation (2.21a) with $h(t) \equiv 1$ is unclear by virtue of the fact that the energy-momentum of Einstein's general theory of relativity is not a tensor. For all the observed redshifts and a flat space the $r - z$ -relations are identical for both gravitational theories. But in the beginning of the universe Einstein's theory yields a singularity, i.e. infinite densities whereas flat space-time theory of gravitation gives non-singular cosmological models, i.e. all the densities are finite.

3. Some Results about Expanding and Non-Expanding Universe

Let us first calculate the energy of a photon emitted by a distant atom at rest and moving to the observer. Let us use the time t with the relations (2.4) and (2.6) and the light velocity (2.18). The energy of a photon emitted by an atom at rest is

$$E = -p_4 c \sim -g_{44} \frac{dx^4}{d\tau}.$$

Hence, it follows for the energy emitted at time t_e by the use of (2.2) with (2.6)

$$E(t_e) = \frac{1}{h(t_e)} \sqrt{h(t_e)} E_0 = \frac{1}{\sqrt{h(t_e)}} E_0 \tag{3.1}$$

where E_0 is the emitted energy of the same atom at present.The energy of the photon moving in the universe follows by the use of the equations of motion (2.3) with $i = 4$, i.e.

$$\frac{d}{dt}\left(-\frac{1}{h(t)} \frac{dt}{d\tau} \right) = \left(a\dot{a} \frac{1}{c^2} \sum_{k=1}^{3} \left(\frac{dx^k}{dt} \right)^2 + \frac{1}{2} \frac{\dot{h}}{h^2} \right) \frac{dt}{d\tau}.$$

Let us substitute the light velocity (2.18) into this equation then we get

$$-\frac{d}{dt}\left(\frac{1}{h(t)} \frac{dt}{d\tau} \right) = \left(\frac{\dot{a}}{a} + \frac{1}{2} \frac{\dot{h}}{h} \right) \frac{1}{h} \frac{dt}{d\tau}.$$

This differential equation has the solution

$$\frac{1}{h} \frac{dt}{d\tau} = \frac{C_0}{a(t)\sqrt{h(t)}}$$

with a constant of integration C_0. Hence, the energy is given by (with a new constant C_1):

$$E(t) = \frac{C_1}{a(t)\sqrt{h(t)}}.$$

We get by the use of (3.1) $C_1 = a(t_e) E_0$ implying the energy

$$E(t) = \frac{a(t_e)}{a(t)\sqrt{h(t)}} E_0. \tag{3.2}$$

Hence, the energy of the photon in the universe decreases with the time and at present:

$$E(0) = a(t_e) E_0. \tag{3.3}$$

Let $(p_1(t), p_2(t), p_3(t), p_4(t))$ denote the four-momentum of the photon in the universe with $p_4(t) = -E(t)/c$ then it follows from (2.2) with (2.6)

$$\frac{1}{a^2(t)} |p(t)|^2 - h(t) p_4{}^2(t) = 0.$$

Here, $|\cdot|$ denotes the Euclidean norm of (p_1, p_2, p_3). Therefore, we get

$$|p(t)| = a(t)\sqrt{h(t)} |p_4(t)| = a(t_e) E_0 / c. \tag{3.4}$$

The equations (3.2) and (3.4) give for the wavelength and the frequency of the photon by the use of Planck's law

$$\lambda(t) = \frac{1}{a(t_e)} \frac{c}{\nu_0}, \qquad \nu(t) = \frac{a(t_e)}{a(t)\sqrt{h(t)}} \nu_0 \tag{3.5}$$

for the moving photon where ν_0 is the frequency emitted by the same atom at present. Hence, the wavelength is constant during the motion of the photon whereas the frequency decreases and it holds:

$$\lambda(t)\nu(t) = c/\left(a(t)\sqrt{h(t)}\right) = |v_l(t)|. \tag{3.6}$$

At present time the observer measures for the arriving photon

$$\lambda(0) = \frac{1}{a(t_e)}\frac{c}{\nu_0}, \qquad \nu(0) = a(t_e)\nu_0, \qquad \lambda(0)\nu(0) = c.$$

Let us now introduce the times $\tilde{t}$ resp. t'. It follows by the use of (2.12a) or (2.15a) and the transformation formulas

$$\tilde{p}_i = p_k \frac{\partial x^k}{\partial \tilde{x}^i}, \qquad p_i' = p_k \frac{\partial x^k}{\partial x^{i\prime}} \qquad (i = 1-4)$$

the result

$$\tilde{p}_i = p_i' = p_i \quad (i = 1,2,3), \qquad \tilde{p}_4 = p_4\sqrt{h}, \qquad p_4' = p_4 a\sqrt{h}.$$

Hence, the relations (3.4) and (3.2) yield:

$$|\tilde{p}(t)| = |p'(t)| = a(t_e)E_0/c,$$

$$\tilde{E}(t) = \frac{a(t_e)}{a(t)}E_0, \quad E'(t) = a(t_e)E_0.$$

The wavelength and the frequency of the photon in the universe are therefore

$$\tilde{\lambda}(t) = \lambda'(t) = \frac{1}{a(t_e)}\frac{c}{\nu_0}, \tilde{\nu}(t) = \frac{a(t_e)}{a(t)}\nu_0, \ \nu'(t) = a(t_e)\nu_0. \tag{3.7}$$

The relations (3.7) yield by the use of (2.18)

$$\tilde{\lambda}(t)\tilde{\nu}(t) = c/a(t) = |\tilde{v}_l(t)|, \qquad \lambda'(t)\nu'(t) = c = |v_l'(t)|. \tag{3.8}$$

Hence, the wavelength of the photon is constant during its motion through the universe independently of the used time whereas the frequency is in general changeing and it is constant only by the use of the observer’s time. These considerations of a non-expanding space are already contained in the paper [12]. In the paper [13] generalizations to a moving body in the universe are considered implying an explanation of the anomalous Doppler frequency shift of the Pioneers. An unpleasent result of cosmological models by Einstein’s theory is the non-conservation of the energy of the photon moving in the universe (see [17]).

Let us now consider the expanding universe with the transformations (2.20) and the corresponding transformation fomulas for the four-momentum. This gives the relations

$$\tilde{p}_i = \frac{1}{a(t)}p_i \quad (i = 1,2,3), \qquad \tilde{p}_4 = -p_k\tilde{x}^k \frac{1}{c}\frac{\dot{a}}{a^2}\sqrt{h} + p_4\sqrt{h}.$$

This implies

$$|\tilde{p}(t)| = \frac{1}{a(t)}|p(t)|, \qquad \tilde{E}(t) = \frac{a(t_e)}{a(t)}E_0 + (p,\tilde{x})\frac{\dot{a}}{a^2}\sqrt{h} \tag{3.9}$$

where $(\cdot,\cdot)$ denotes the scalar product. Let us assume that the momentum of the photon is opposite to the line of sight, i.e. the photon moves to the observer then we get

$$\tilde{E}(t) = \frac{a(t_e)}{a(t)}E_0 - |p||\tilde{x}|\frac{\dot{a}}{a^2}\sqrt{h}.$$

This relation gives for the energy of the photon:

$$\tilde{E}(t) = \frac{a(t_e)}{a(t)}E_0\left(1 - \frac{1}{c}\frac{\dot{a}(t)}{a(t)}\sqrt{h(t)}|\tilde{x}|\right). \tag{3.10}$$

Hence, the wavelength and the frequency of the photon in the expanding universe are

$$\tilde{\lambda}(t) = \frac{a(t)}{a(t_e)}\frac{c}{\nu_0}, \qquad \tilde{\nu}(t) = \frac{a(t_e)}{a(t)}\nu_0\left(1 - \frac{1}{c}\frac{\dot{a}(t)}{a(t)}\sqrt{h(t)}|\tilde{x}|\right) \tag{3.11}$$

and by virtue of (2.23)

$$\tilde{\lambda}(t)\tilde{\nu}(t)=c\left(1-\frac{1}{c}\frac{\dot{a}(t)}{a(t)}\sqrt{h(t)}|\tilde{x}|\right)=|\tilde{v}_l(t)|. \tag{3.12}$$

It follows that the wavelength is increasing and starts from the wavelength of the observer at present.The increasing wavelength in the universe is the well-known argument for the expansion of space whereas the frequency of the moving photon (3.11) is very complicated. At present time all the results of the wavelength and the frequency are identical independently of the used time $t,\tilde{t}$ or t' or an expanding universe. It is worth mentioning that the results for the wavelength and frequency using the time $\tilde{t}$ or t' or the expanding space can also be received in analogy to the corresponding considerations as with the time t but these calculations are more complicated.

Let us now consider the total energy-momentum of the universe. It is given using the time t by equation (2.10). We consider the transformation formulas (2.12a), (2.15a) or (2.20) using the time $\tilde{t}\,,t'$ or the expansion of space together with the formula for the transformation of tensors given by

$$\tilde{T}^i{}_j=T^k{}_l\frac{\partial\tilde{x}^i}{\partial x^k}\frac{\partial x^l}{\partial\tilde{x}^j}.$$

Then, we get for the times $\tilde{t}$ and t'

$$\tilde{T}^i{}_j=T^i{}_j{}'=T^i{}_j \qquad (i,j=1,2,3,4) \tag{3.13}$$

and for an expanding universe

$$\begin{aligned}&\tilde{T}^i{}_j=T^i{}_j, \quad T^4{}_j=0 \quad (i,j=1,2,3)\\ &\tilde{T}^i{}_4=-\frac{1}{c}\frac{\dot{a}}{a}\sqrt{h}\left(\sum_{k=1}^{3}\tilde{x}^kT^i{}_k-\tilde{x}^iT^4{}_4\right) \qquad (i=1,2,3)\\ &\tilde{T}^4{}_4=T^4{}_4\ .\end{aligned} \tag{3.14}$$

Hence, we have by virtue of (3.13) and (3.14) and the conservation of of the total energy of the universe using the time t that the total energy is always conserved independently of the used time or an expansion of space.

The conservation of the total energy in an expanding space can also directly proved by the conservation law of energy-momentum for $i=1-4$:

$$\tilde{T}^k{}_{i;k}=\frac{\partial\tilde{T}^k{}_i}{\partial x^k}+\Gamma^k{}_{kl}\tilde{T}^l{}_i-\Gamma^k{}_{il}\tilde{T}^l{}_k=0$$

where $\Gamma^i{}_{jk}$ are the Christoffel symbols of the flat space-time metric (2.21b).The calculations are longer and are omitted here.

Einstein's theory implies for a flat space that the total energy of a non-expanding universe is identical zero but it is worth mentioning that the energy-momentum of Einstein's theory is only a pseudo-tensor (see [14] and [18]).

Next, let us consider the motion of a test particle in the gravitational field of a solid body in the universe. The post-Newtonian approximation of the equations of motion of several bodies in the universe is studied in paper [15] . In the special case of a body at rest with mass M and a test particle with velocity $\left(\frac{dx^1}{dt'},\frac{dx^2}{dt'},\frac{dx^3}{dt'}\right)$ where the observer's time is used the equations of motion have the form:

$$\frac{d}{dt'}\left(a\frac{dx^i}{dt'}\right)=-kM\frac{x^i}{|x|^3} \quad (i=1,2,3). \tag{3.15}$$

Elementary calculations give

$$\frac{d^2}{dt'^2}\left(ax^i\right)-\frac{1}{a}\frac{da}{dt'}\frac{d\left(ax^i\right)}{dt'}-\frac{d}{dt'}\left(\frac{1}{a}\frac{da}{dt'}\right)\left(ax^i\right)=-a^2kM\frac{ax^i}{|ax|^3}.$$

Put

$$\tilde{x}^i=ax^i \quad (i=1,2,3) \tag{3.16}$$

then, it follows

$$\frac{d^2\tilde{x}^i}{dt'^2}-\frac{1}{a}\frac{da}{dt'}\frac{d\tilde{x}^i}{dt'}-\frac{d}{dt'}\left(\frac{1}{a}\frac{da}{dt'}\right)\tilde{x}^i=-a^2kM\frac{\tilde{x}^i}{|\tilde{x}|^3} \quad (i=1,2,3).$$

At present time we get by neglecting terms of order $O\left(H_0{}^2\right)$:

$$\frac{d^2\tilde{x}^i}{dt'^2} - H_0\frac{d\tilde{x}^i}{dt'} = -a^2kM\frac{\tilde{x}^i}{\left|\tilde{x}\right|^3} \qquad (i = 1,2,3). \tag{3.17}$$

Put

$$\tilde{x}^1 = \tilde{r}\cos\varphi, \quad \tilde{x}^2 = \tilde{r}\sin\varphi, \quad \tilde{x}^3 = 0$$

then two differential equations for $\tilde{r}$ and φ are received:

$$-2\frac{d\tilde{r}}{dt'}\frac{d\varphi}{dt'} - \tilde{r}\frac{d^2\varphi}{dt'^2} + H_0\tilde{r}\frac{d\varphi}{dt'} = 0$$

$$\frac{d^2\tilde{r}}{dt'^2} - \tilde{r}\left(\frac{d\varphi}{dt'}\right)^2 - H_0\frac{d\tilde{r}}{dt'} = -\left(1 + H_0\left(t' - t_0'\right)\right)^2\frac{kM}{\tilde{r}^2}$$

where t_0' denotes the observer’s time at present. The first equation is solved with the solution

$$\tilde{r}^2\frac{d\varphi}{dt'} = C\exp\left(H_0\left(t' - t_0'\right)\right) \tag{3.18}$$

where C is a constant of integration. The substitution of (3.18) into the second differential equation gives:

$$\frac{d^2\tilde{r}}{dt'^2} - H_0\frac{d\tilde{r}}{dt'} = C^2\frac{1}{\tilde{r}^3}\left(1 + 2H_0\left(t' - t_0'\right)\right) - \left(1 + 2H_0\left(t' - t_0'\right)\right)\frac{kM}{\tilde{r}^2}.$$

Assuming that the test body is moving on a perturbed sphere with radius $\tilde{r}_0$ and let $\tilde{r} = \tilde{r}_0 + \Delta\tilde{r}$ then the above differential equation gives

$$C^2 = kM\tilde{r}_0, \qquad \frac{d^2\Delta\tilde{r}}{dt'^2} - H_0\frac{d\Delta\tilde{r}}{dt'} = -\frac{kM}{\tilde{r}_0^3}\Delta\tilde{r}.$$

The last equation implies the solution $\Delta\tilde{r} = 0$. Hence, we have the constant solution $\tilde{r}_0$:

$$\tilde{r}_0 = \tilde{r} = ar(t'). \tag{3.19}$$

This relation gives:

$$r(t') = \frac{1}{a}\tilde{r}_0 \approx \left(1 - H_0\left(t' - t_0'\right)\right)\tilde{r}_0. \tag{3.20}$$

In a non-expanding universe relation (3.20) states that the test body is spirally moving towards the solid body whereas in an expanding space relation (3.19) implies that the test body moves on a fixed sphere. This last result is generally formulated as a bound system in the expanding universe does not expand (see [16]). Both results about an expanding and a non-expanding space are not in agreement with the experimental results that the moon is moving away from the earth. It is worth mentioning that the above study about a test body in the gravitational field of a solid body at rest in the universe is too simple to describe the earth-moon system. Further effects has to be taken into consideration such as the rotation of the earth, tidal effects, other planets, etc..

Summarizing, we can say that the studied results give no definite answer whether space is expanding or not. independently of the used theory of gravitation. It is worth mentioning that Einstein’s theory gives singular cosmological models and the energy-momentum complex is not a tensor in contrast to flat space-time theory of gravitation. Einstein’s theory suggests an expanding space by virtue of the definition of the line-element (2.2) with (2.13) resp. (2.21a) whereas flat space-time theory suggests a non-expanding space by virtue of the background metric (2.1) with (2.4) or (2.14) or (2.17) whereas (2.2) with (2.6) or (2.13) or (2.16) defines the proper time of the universe.

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