On the Planck Scale and Structures of Matter

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Abstract

Invariant properties and structures of matter are modeled by internal period-like degrees of freedom. Invariance then means periods, which remain unaltered over time. Period doubling is a phenomenon common to nonlinear dynamical systems. In this model the doubling process is generalized into multiple dimensions and utilized to bring about sub-harmonic frequencies, which generate decreasing energies and increasing sizes. It is assumed that period doubling takes place at the Planck scale, and therefore the Planck units are used as reference. The sub-harmonics can be converted into several other physical quantities by well known physical relations. A certain class of sub-harmonics is stable and the elementary electric charge (squared), rest energies and magnetic moments of the electron-positron and proton-antiproton pairs are shown to belong to this class. It is suggested that the structure of the Solar system results from period doubling, too.

List of symbols

a	Constant
A	Area
α	Fine structure constant
c	Speed of light
E	Energy
E_o	Planck energy for the electron
E_{op}	Planck energy for the proton
ε_o	Electric constant
f	Frequency
F	Force
G	Gravitational constant
h	Planck's constant
i	Electric current
i,j,k,l	Integers
i_o	Planck current
l_o	Planck length
μ	Magnetic moment
μ_o	Planck scale magnetic moment for the electron
μ_{op}	Planck scale magnetic moment for the proton
n	Perceived number of doublings
N	Total number of doublings
q_o	Planck charge
r	Radius or distance
τ	Period
τ_o	Planck time (period)
v	Velocity
V	Potential

1 Introduction

Period doubling is a universal property of nonlinear dynamical systems [1], which means that a series of sub-harmonic frequencies is borne within the system. If the Planck time is chosen as unit period, then the sub-harmonic frequencies are fixed and unadjustable. Periods and frequencies can be converted into other physical quantities, which then exhibit the period doubling structure, too.

Because real world objects are multi-dimensional, we will generalize period doubling into multiple dimensions, which will be utilized as mutually independent internal degrees of freedom of the system in consideration. By a system we mean a naturally occurring group of objects or phenomena. The objects may be internal structures of the elementary particles or the planets in the Solar system. Systems under the common nonlinear gravitational and Coulomb potentials are studied in this work.

We shall show that many of the calculated values are coincident with observation, especially those belonging to a sub-class of periods, which are stable.

The objective of this article is to introduce the basic concepts of the model published in [2].

2 Motivation

The objective of this work is to find a connection between the Planck scale defined by natural constants and the stationary properties and structures of matter.

3 Assumptions

1. A nonlinear dynamical system, acting at the Planck scale, generates sub-harmonic frequencies by multidimensional period doubling. Periods serve as internal degrees of freedom.
2. The periods, or sub-harmonic frequencies, can be related to other physical quantities by known relations.

4 Parameters

The number of doublings in each degree of freedom serves as an integer parameter.

5 Model

We model the intrinsic properties of a dynamical system with mutually independent internal degrees of freedom, which can be described by constant periods of revolution. By a system we mean a naturally occurring group of objects or phenomena, e.g. planets in the Solar system or the internal structures of the elementary particles

Such a system is shown in Fig. 1, where the "sphere" is the system under consideration and (x',y',z') are the axes of internal rotation. Without external influence (e.g. magnetic field) the directions of the (x',y',z')-axes are arbitrary with respect to the spatial directions (x,y,z). The system itself is free to move in space.

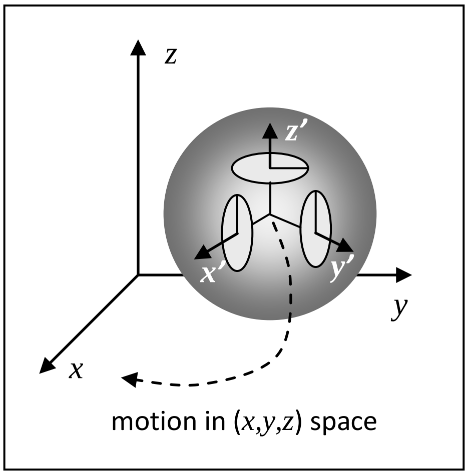

Fig. 1. Internal degrees of freedom, or periods, describe the stationary properties of the system.

Several physical quantities can be represented in terms of period. The well known Planck relation for energy is $E=hf=h/\tau$, showing that energy is inversely proportional to period. Lengths can be calculated from $l=ct$ and magnetic moments from $\mu=iA$. If the system under consideration has a fundamental period and corresponding energy, then period doubling, i.e. sequential multiplication by 2, brings about a series of increasing periods

$$\tau_N = 2^N \tau_o \qquad (1)$$

and decreasing energies:

$$E_N = \frac{h}{\tau_N} = \frac{h}{2^N \tau_o} = 2^{-N} E_o \qquad (2)$$

The Planck time (considered as the fundamental period) is $\tau_o \approx 1.35 \cdot 10^{-43}$ s and it can be calculated from the Planck mass $m_o=(hc/G)^{1/2}$. The Planck energy is $E_o=h/\tau_o \approx 3.06 \cdot 10^{22}$ MeV and the unit electric charge $q_o=(4\pi\varepsilon_o hc)^{1/2} \approx 4.70 \cdot 10^{-18}$ As.

6 Generalization of period doubling

Real objects, like an atom, are three dimensional. We may model these objects by multiple internal degrees of freedom, i.e. by rotational periods τ_i, τ_j and τ_k about (x',y',z')-axes in three dimensions. The corresponding periodic volume is then

$$V_{ijk}(\tau) = \tau_i \tau_j \tau_k \qquad (3)$$

In four degrees of freedom one obtains correspondingly

$$V_{ijkl}(\tau) = \tau_i \tau_j \tau_k \tau_l \qquad (3')$$

Eq.'s (3) and (3') define both *shape* and *size*, and period doubling manifests as *volumetric doubling*. The internal structures (3) and (3') can be considered as parallelepipeds or ellipsoids, the geometry of which is determined by i, j, k and l , all integers. The total number of period doublings is $i+j+k$ or $i+j+k+l$. Combining (1) and (3) or (3') yields

$$V_{ijk}(\tau) = 2^i \tau_o \cdot 2^j \tau_o \cdot 2^k \tau_o = 2^{i+j+k} \tau_o^{\ 3} \qquad (4)$$

$$V_{ijkl}(\tau) = 2^i \tau_o \cdot 2^j \tau_o \cdot 2^k \tau_o \cdot 2^l \tau_o = 2^{i+j+k+l} \tau_o^{\ 4} \qquad (4')$$

Equations (4) and (4') can be written in terms of energy:

$$E_{ijk}^{\ 3} = \frac{h^3}{2^{i+j+k} \tau_o^{\ 3}} = 2^{-(i+j+k)} E_o^{\ 3} \qquad (5)$$

and correspondingly for four degrees of freedom. Note that (5) also tells the distribution of the internal energy. The perceived energies are obtained from (5) by taking cube root:

$$E_{ijk} = 2^{-\frac{i+j+k}{3}} E_o \tag{6}$$

A fourth root is taken for four degrees of freedom ($i+j+k+l$). When the model is compared with measurement E_{ijk} or E_{ijkl} is replaced with the measured energy in (4) or (4'). The total number of doublings can be solved for from

$$i+j+k = \frac{\log(\frac{E_o}{E_{ijk}})}{\log(2)} \cdot 3 \tag{7}$$

$$i+j+k+l = \frac{\log(\frac{E_o}{E_{ijkl}})}{\log(2)} \cdot 4 \tag{7'}$$

For the model to be successful the total number of doublings, i.e. $i+j+k$ or $i+j+k+l$ should be integers when comparison with experiment is made.

7 Theory

One of the most interesting nonlinear potentials is $V=-1/r$, whose gradient is the well known Coulomb and gravitational force $F=1/r^2$. A second order differential equation can be derived by combining the $1/r$ potential and $r^3=const \cdot \tau^2$ (spatial volume proportional to period squared, Kepler's third law):

$$\frac{d^2 r}{d\tau^2} = -\frac{a}{\tau^2} r \tag{8}$$

The solution of (8), shown in Fig. 2, possesses three- and four dimensional (i.e. volumetric) doubling behavior, depending on the value of constant a [2]. a=46.47 doubles the 3-d volume.

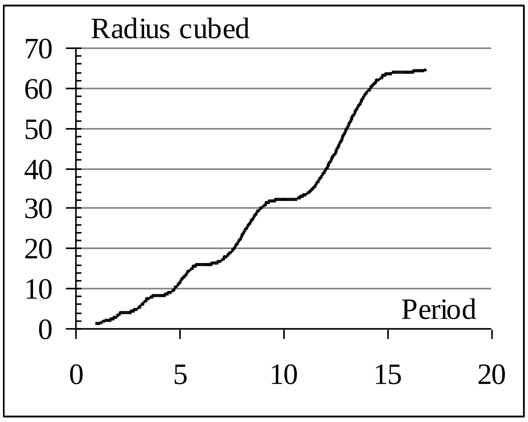

Fig. 2. 3-d volumetric doubling in (8).

8 Condition for stability

The total number of doublings can, in principle, be any integer. However, only a few stable structures of matter exist. Therefore there must be a special condition for the stable doublings. It is shown in [2] that doublings of the form (9) are stable, if m is an integer.

$$\tau_m = 2^{2^m} \tau_o \tag{9}$$

9 Experimental

In the next chapters we shall take a look at some fundamental properties of matter, and calculate the total number of doublings and show that they are integers.

9.1 Elementary electric charge

The Planck scale electric charge (squared) is defined

$$q_o^{\ 2} = 4\pi\varepsilon_o hc \tag{10}$$

The numeric value of the charge is q_o=4.7013·10^{-18} As, surprisingly close to the elementary electric charge e=1.602·10^{-19} As. This means that only a few doublings are needed for obtaining the elementary electric charge from the Planck charge. Because energy is directly proportional to charge squared (and inversely proportional to period), we shall calculate the number of doublings for the ratio of the charges squared:

$$\frac{\log(\frac{q_o^{\ 2}}{e^2})}{\log(2)} = 9.750 = \frac{39}{4} \tag{11}$$

Number 4 in the denominator means that the number of internal degrees of freedom is four. The total number of doublings is $i+j+k+l$=39.00 and we perceive the fourth root instead of the cube root, as before. It means that the electrostatic energy of the object has a 4-d shape, and must be described with four degrees of freedom contrary to rest mass-energy, where three degrees of freedom are sufficient (as shown before). Equation (12) clarifies the situation with the elementary electric charge squared.

$$e^2 = 2^{-\frac{i+j+k+l}{4}} q_o^{\ 2} = 2^{-\frac{39}{4}} q_o^{\ 2} \tag{12}$$

The numeric value of the elementary electric charge can be calculated from (12), yielding e=1.60213·10^{-19} As, which differs from the accepted value of e by 0.003% [3].

9.2 Fine structure constant

The fine structure constant is defined

$$\alpha = \frac{e^2}{2\varepsilon_o hc} = 2\pi \frac{e^2}{q_o^{\ 2}} = 2\pi \cdot 2^{-\frac{39}{4}} \tag{13}$$

where (10) has been utilized. The numeric value for $1/\alpha$ is 137.045, which differs from the present accepted value (137.036, [3]) by 0.007%. Equation (13) shows that the fine structure constant is a pure number resulting from the ratio $(e/q_o)^2$ and period doubling in four degrees of freedom.

9.3 Electron-positron pair

9.3.1 Rest energy

An electron and a positron can be created simultaneously from a gamma quantum in the pair production process. The rest energy of the pair is 1.022 MeV and (7) yields $i+j+k$=224.00 for the total number of doublings.

9.3.2 Magnetic moment

Magnetic moment of a current loop is classically defined as current times area, or $\mu=iA$. The Planck scale magnetic moment will be defined as shown in Fig. 3. The circumference of the current loop is, in the Bohr sense, one Planck length $l_o=c\tau_o$, hence the radius is $l_o/2\pi$. If an elementary charge e is orbiting the loop in the Planck time τ_o, then the current is $i=e/\tau_o$. The Planck scale magnetic moment is then

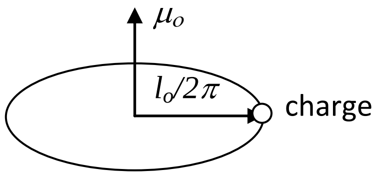

Fig. 3. Definition of the Planck scale magnetic moment.

$$\mu_o = iA = \frac{e}{\tau_o}\pi(\frac{l_o}{2\pi})^2 = \frac{1}{4\pi}ec^2\tau_o \tag{14}$$

where $A=\pi r^2=\pi(l_o/2\pi)^2$. Note that the magnetic moment is directly proportional to period. The numerical value of the unit magnetic moment is μ_o=3.8208·10^{-46} Am^2.

The magnetic moment of an electron-positron pair can be modeled by two current loops, where the positive and negative elementary charges revolve in anti-phase, or opposite directions. The sum of the magnetic moments is thus half the value of an electron alone (because $\tau_e=2\tau_{ep}$).

Because magnetic moment is directly proportional to period in (14), one can write

$$\mu_{ijk} = \frac{ec^2}{4\pi}\cdot 2^{\frac{i+j+k}{3}}\tau_o = 2^{\frac{i+j+k}{3}}\mu_o \tag{15}$$

The measured (absolute) value of the magnetic moment of an electron is μ=9.285·10^{-24} Am^2, and half for the electron-positron pair. The number of magnetic moment doublings can be calculated from (15) by inserting the corresponding values. One obtains $N=i+j+k$=224.00, which is the same as what was obtained for the rest energy of the electron-positron (e-p) pair. The results are summarized in Table I (paragraph 10). A more detailed analysis is carried out in [2].

9.4 Proton

The electron is described as a structureless pointlike particle, whereas the proton has a measurable size and internal structure. In modeling the proton a clear distinction between the electron and the proton structures must be made.

9.4.1 Rest energy

The original Planck energy was taken as the unit energy for the electron, but in order to make the proton a composite particle we shall choose half of the sum energy of two adjacent energy levels, E_o and $2^{0.3333}E_o$, for the unit energy. Note that adjacent levels are separated by a factor of cube root of two in three dimensions.

$$E_{op} = ½(1+2^{0.3333})E_o \tag{16}$$

The numeric value of E_{op} is 3.4580·10^{22} MeV. Equation (7) yields $i+j+k$=192.00 for the proton-antiproton pair (2*928 MeV). A more detailed calculation is shown in [2].

9.4.2 Magnetic moment

A Bohr-type current loop was defined for the electron. The Planck scale radial type current loop for the proton is shown in Fig. 4. The diameter of the loop is half of the Planck length.

Half wave diameter corresponds to the ground state of a potential well. The value of the unit magnetic moment for the geometry in Fig. 4 is

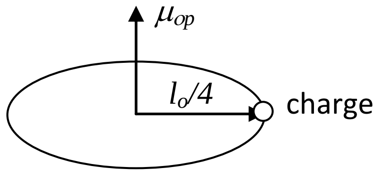

Fig. 4. Definition of the Planck scale magnetic moment for the proton.

$$\mu_{op} = iA = \frac{e}{\tau_o}\pi(\frac{l_o}{4})^2 = \frac{\pi}{16}ec^2\tau_o \qquad (17)$$

The numeric value is μ_{op}=3.8208·10^{-46} Am^2. The magnetic moment of the proton is 1.4106·10^{-26} Am^2, twice the proton-antiproton pair. Eq. (17) yields

$$\mu_{ijk} = \frac{\pi ec^2}{16}\cdot 2^{\frac{i+j+k}{3}}\tau_o = 2^{\frac{i+j+k}{3}}\mu_{op} \qquad (18)$$

The number of doublings for the proton-antiproton pair is $i+j+k$=192.00 from (18). Table I shows that rest energy and magnetic moment belong to the same individual structure, which develops from the Planck scale by period doubling. The total number of doubling is 224.00 for the electron-positron pair and 192.00 for the proton-antiproton pair.

9.5 The Solar system

Period doubling from the Planck scale generates increasing periods and lengths ($l=c\tau$). Velocities v can be defined as follows:

$$v_n = \frac{2^i l_o}{2^j \tau_o} = 2^{i-j}c = 2^{-n}c \qquad (19)$$

where n is the perceived number of doublings. Figure 5 shows the calculated and observed orbital velocities (at semimajor axis) and semimajor axes of the orbits of the planets. The total number of doublings, i.e. 3-d velocity halvings, is $N=3n=i+j+k$, because the n-values, calculated from (19), are of the form $N/3$ (i.e. cube root).

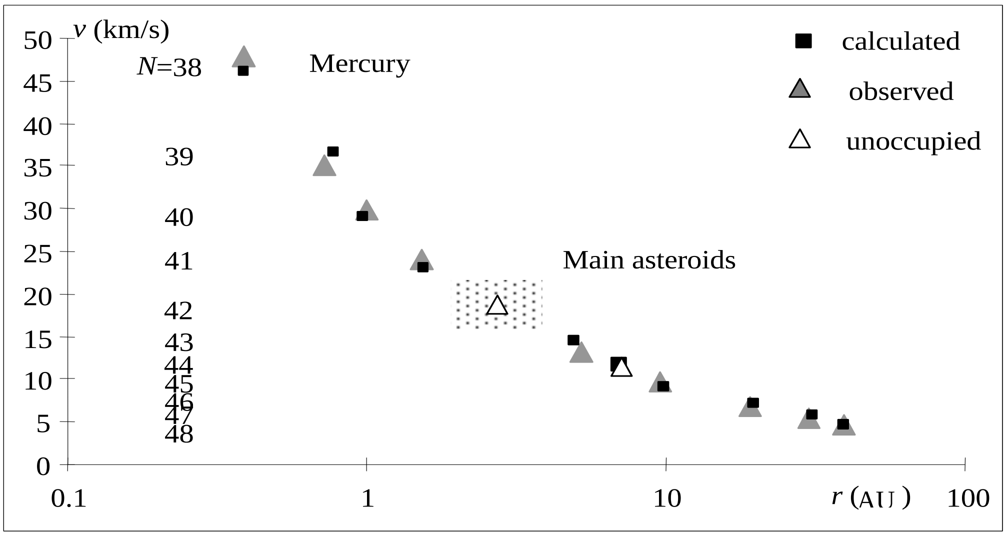

Fig. 5. Planets in (r,v) space. The asteroids occupy space around $N=3n=42$ orbit and $N=44$ is empty. Orbits of $N=42$ and $N=44$ are on both sides of Jupiter ($N=43$). The N-values indicated belong to v in (19).

It should be noted that the N-values in Fig. 5 are consequent integers suggesting that the primordial matter around the Sun accumulated in places determined by the period doubling process. The radii of the orbits are calculated from

$$r_m = 2^{\frac{M}{3}} r_o \qquad (20)$$

Table I. Stability condition.

Object	Total	nr. doublings
		$i+j+k+l$
Elementary charge squared	39	1+2+4+32
Electron-positron pair		$i+j+k$
Rest energy	224	32+64+128
Magnetic moment	224	32+64+128
Proton-antiproton pair		
Rest energy	192	64+64+64
Magnetic moment	192	64+64+64

10 Stability of the basic structures

In the examples above $i+j+k$=224 was obtained for the electron-positron pair and $i+j+k$=192 for the proton-antiproton pair. If $i+j+k$ is written in terms of the internal degrees of freedom, one finds that

$i+j+k$=224=32+64+128=$2^5+2^6+2^7$

demonstrating the stability condition (9) for *each degree of freedom*. Similarly $i+j+k$=192= 64+64+64=$2^6+2^6+2^6$. For the elementary electric charge squared (≈energy) 39=$i+j+k+l$=1+2+4+32= $2^0+2^1+2^2+2^5$, all conforming to the stability requirement.

11 Discussion

A simple statistical analysis for a larger sample of experimental values is carried out in [2] showing that the total number of doublings tends to group around an integer value.

The unit energy (in 9.4.1) for the proton-antiproton pair is chosen such that the total number of doublings is the same for the rest energy and magnetic moment, which are properties of the same particle.

The magnetic moment of a current loop depends on the size of the loop. We may therefore assume, based on the good agreement between the calculation and measurement, that the internal degrees of freedom are not purely mathematical presentations, but reflect the real physical size of the object. This assumption is further corroborated by the calculated “size” of a proton, namely $2^{65}\cdot(l_o/2)$=0.7 fm, in agreement with observation. We may therefore assume that the fourth degree of freedom needed in the description of the internal electric energy of the object has real spatial nature, too. A metric fourth dimension is also used by Suntola [4, 5] in modeling electromagnetic dipole radiation from an object much smaller than the emitted wavelength.

12 Summary

Stationary properties of matter are modeled by periods, which serve as internal degrees of freedom. Period doubling, taking place in nonlinear dynamical systems, is used to generate structure to the system in consideration. The required number of internal degrees of freedom is three or four depending on the property under consideration. It seems that the degrees of freedom, or dimensions, are related to the physical size of the object.

The Planck units, defined by natural constants, are chosen as reference. The calculated values are fixed and, in principle, unadjustable. The examples show that the total number of doublings is very close to an integer, which is the requirement for period doubling to occur. The stability condition is fulfilled, too.

13 Scientific value of the work

The work connects period doubling, the universal property of nonlinear dynamical systems, to the natural constants and the observable world in a simple way. The model is applicable to a wide range of stationary properties and structures of matter.

The work opens a possibility to analyze the elementary particles in view of different geometries generated by the doubling process, and sum and difference energies obtained from the Planck sub-energies. Violent collisions may generate electric charges other than the elementary electric charge according to (12). Particle masses, determined by assuming the elementary electric charge, would change correspondingly.

Period tripling may occur simultaneously with period doubling and be responsible for some stationary properties of matter. This is one possible direction for further studies, too.

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